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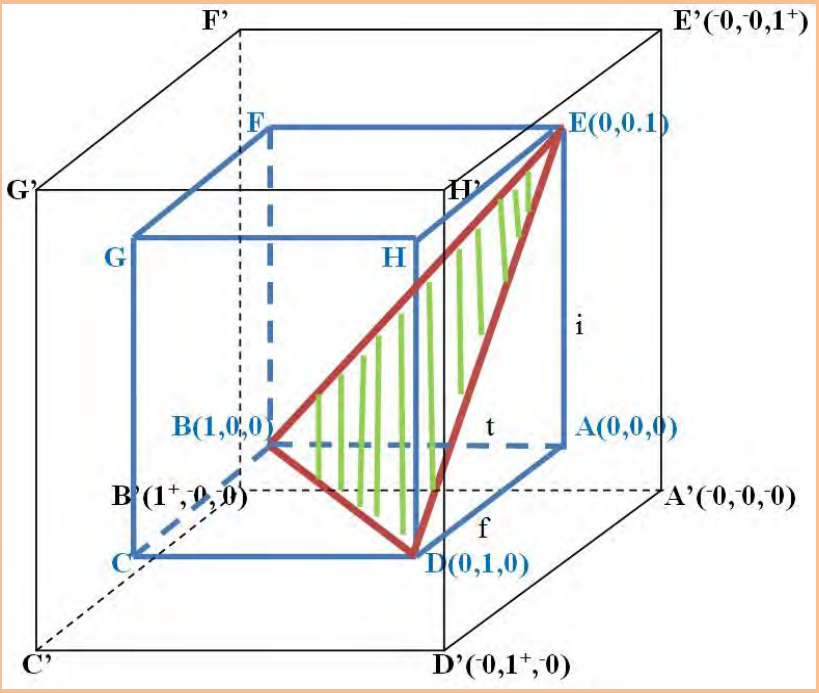

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The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a degree of indeter

minacy (*I*), and a degree of falsity (*F*), where *T, I, F* are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Interpreting and Expanding Confucius' Golden Mean through Neutrosophic Tetrad

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Abstract. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. There are many similarities between The Golden Mean and Neutrosophy. Chinese and international schol-ars need to toil towards expanding and developing The Golden Mean, towards its "modernization" and "globali-zation". Not only Chinese contemporary popular ideas and methods, but also international contemporary popular ideas and methods, should be applied in this endeavour. There are many different ways for interpreting and expanding The Golden Mean through "Neutrosophic tetrad" (thesis-antithesis-neutrothesis-neutrosynthesis). This paper em-phasizes that, in practice, The Golden Mean cannot be applied alone and unaided for long-term; it needs to be combined with other principles.

Keywords: Neutrosophy, Golden Mean, Neutrosophic tetrad, thesis-antithesis-neutrothesis-neutrosynthesis.

1 Introduction

"The Golden Mean" is a significant achievement of Confucius (Kong Zi). Mao Zedong considered that Kong Zi's notion of Golden Mean is his greatest discovery, and also an important philosophy category, worth discussing over and over.

As well-known, the moderate views originated in ancient times. According to historical records, as the Chinese Duke of Zhou asked Jizi for advice, Jizi presented nine governing strategies, including the viewpoint of "The mean principle". That is the unbiased political philosophy dominated by the upright, and a comprehensive pattern obtained by combining rigidity and moderation. According to the interpretation of many predecessors' viewpoints of "The mean principle", after expanding and developing these viewpoints, Confucius created "The Golden Mean".

After Confucius, many scholars tried to use different ways to interpret and expand "The Golden Mean".

From Tang dynasty, a number of "Neo-Confucianists" emerged, highlighting various characteristics by jointing Confucianism with Buddhism, Daoism, and the like, including Western academic thoughts, and forming numerous new schools.

If regarding "The Golden Mean" presented by Confucius as the first milestone, the thought of "worry before the people and enjoy after the people" produced by Fan Zhongyan, the Chinese Northern Song dynasty's famous thinker, statesman, strategist and writer, can be considered the second milestone of "The Golden Mean". Its meaning is as it follows: neither worry everything, nor enjoy everything; take the middle, namely worry in some cases (before the people), and enjoy in some cases (after the people). This famous saying is perhaps the most meaningful "golden mean". Someone once pointed out that "The Golden Mean" is very conservative, and very negative. However, carefully reading this sentence of Fan Zhongyan, one may adduce a new assessment for "The Golden Mean".

The thought of "traditional Chinese values aided with modern Western ideology" appeared in late Qing Dynasty, and it is the third milestone of "The Golden Mean". Since the first opium war (June, 1840 - August, 1842), in view of the fact that China repeatedly failed miserably in front of the Western powers, some ideologists argued that China must be reformed. The early reformists proposed "the policy mainly governed by Chinese tradition, supported by Western thoughts" (this is also a "middle way"), with the purpose to encourage people to learn from the West, and to oppose against obstinacy and conservatism. In late 19th century, there was a harsh dispute between old and new, and between Chinese model and Western model. The old-fashioned feudal diehards firmly opposed to Western culture. They regarded anything coming from the Western capitalist countries as dangerous evils for China, while the bourgeois reformers actively advocated for Western learning, arguing that China should not only inure the advanced science and technology, but also follow the Western political system. Among the violent debate, ostensibly neutral thought of "Chinese learning for the essence, western learning for practical use" gradually gained prominence, and had a profound impact. Even today, there still exist scholars appraising this slogan, and attempting to make new interpretations out of it.

But "The Golden Mean" is not uniquely Chinese. One can also find similar formulations in different cultures. For example, in Ancient Greece, Aristotle propounded the idea of "The Mean Principle". According to Aristotle, there are three categories of human acts, namely excessive, less and moderate acts. For instance, all men have desires; while

excessive desires and less desires are all tidal waves of evil, only moderate desires are virtue-based. There is a clear distance between Aristotle's middle path view and Confucius' Golden Mean, since the last one takes "benevolence" as its core.

In 1995, the American-Romanian scholar Florentin Smarandache created Neutrosophy, which has similarities to "The Golden Mean". For more information about Neutrosophy, see references [1-3].

To sum up, the ideas of "The Golden Mean" and of some similar concepts are crystallizations of mankind wisdom. However, in order to keep pace with the times, "The Golden Mean" and the similar concepts must be expanded and developed in the directions of "modernization" and "globalization". In order to achieve this task, Chinese and international scholars should take part in related actions, and not only Chinese contemporary popular ideas and methods, but also international contemporary popular ideas and methods should be applied. In this way, the results can be widely recognized all over the world, and have a positive and far-reaching impact.

The requisite to expand and develop "The Golden Mean" applying international contemporary popular ideas and methods has not yet attracted enough attention. Consequently, we try to interpret and expand "The Golden Mean" through Neutrosophy, hoping that other scholars will pay attention too to the issues we expound.

2 The similarities between "The Golden Mean" and "Neutrosophy"

In references [2,3] we have pointed out that the position of “mean” pursued by The Golden Mean is the optimized and critical third position, situated between the excessive and the less.

It needs to stress that, according to the fact that Confucius made a great contribution for the amendment of “The Book of Changes”, some people thought that The Analects of Confucius only discussed two kind of situations, i.e. positive and negative situations (masculine and feminine, yin and yang, pro and con), while in fact The Analects evaluated three kind of situations: positive, negative and neutral situations.

For example, in Book 2, Tzu Kung put forward a positive and a negative situation: “What do you pronounce concerning a poor man who doesn't grovel, and a rich man who isn't proud?” Confucius presented the best situation: “They are good, but not as good as a poor man who is satisfied and a rich man who loves the rules of propriety.”

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics, used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic (NL) is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[without necessarily connection between them.

It is obvious that, in discussing the “mean”, the “middle”, or the “neutralities”, there are many similarities between The Golden Mean and Neutrosophy.

It should be mentioned that the biggest difference between Neutrosophy and The Golden Mean is that the first includes a wide variety of practical mathematical methods. Because of some reasons, the mathematical knowledge of many Confucian scholars is not too elevated. Therefore, in general, the Confucian scholars cannot propose quantitative standards to evaluate The Golden Mean, and they only rely on their perception. Nevertheless, Karl Marx believed that a science can only achieve a perfect situation when it is successfully applied to mathematics.

Now we present a simple example of mathematical method application. Let us consider the middle situation composed by "positive" and "negative". The proportion of positive and negative, besides the standard 5:5, also can be 6:4 or 4:6, 7:3 or 3:7, 8:2 or 2:8, 9:1 or 1:9, and so on. For more complex cases, it is necessary to apply the mathematical methods of Neutrosophy.

Therefore, if we need to take into account quantitative relationships, then the mathematical methods of Neutrosophy are helpful. This is one important part of interpreting and expanding "The Golden Mean". Of course, this kind of work need to be undertaken by scholars who are familiar with both "The Golden Mean" and "Neutrosophy".

3 Interpreting and expanding The Golden Mean with “Neutrosophic tetrad” (thesis-antithesis-neutrothesis-neutrosynthesis)

In reference [4], Prof. Smarandache called attention for the fact that the classical reasoning development about evidences, popularly known as thesis-antithesis-synthesis from dialectics, was attributed to the renowned philosopher Georg Wilhelm Friedrich Hegel, and it was used later on by Karl Marx and Friedrich Engels. Immanuel Kant have also written about thesis and antithesis. As a difference, the opposites yin [feminine, the moon] and yang [masculine,

the sun] were considered complementary in Ancient Chinese philosophy.

Neutrosophy is a generalization of dialectics. Therefore, Hegel's dialectical triad thesis-antithesis-synthesis is extended to the neutrosophic tetrad thesis-antithesis-neutrothesis-neutrosynthesis. A neutrosophic synthesis (neutrosynthesis) is more refined that the dialectical synthesis. It carries on the unification and synthesis regarding the opposites, and their neutrals too.

There are many different ways for interpreting and expanding The Golden Mean through "Neutrosophic tetrad" (thesis-antithesis-neutrothesis-neutrosynthesis), and different conclusions are reached. This paper emphasizes the conclusion that, in practice, The Golden Mean cannot be applied lonely and unaided for long-term; in many cases, it needs to be combined with other principles.

Example 1: If asking a man who likes to do everything according to The Golden Mean: will you wear black or white clothes to attend the meeting?, the answer should be an unbiased one: I will wear grey clothes. According to "Neutrosophic tetrad" (thesis-antithesis-neutrothesis-neutrosynthesis), there are many different possible answers: (1) I will wear deep grey clothes; (2) I will wear shallow grey clothes; (3) I will wear a white coat, but black trousers; (4) trousers white underwear, but a black coat; (5) I will wear black clothes at the beginning of the meeting, but white clothes at the end of the meeting; (6) I will switch between black, grey, and white clothes during the meeting; (7) I will wear black clothes at this conference, but white clothes at the next one; (8) I will respectively wear black, white, grey (or different combination of the three colours) clothes at different conferences. And so forth.

In this example, The Golden Mean cannot be applied lonely and unaided for long-term; in fact, no one can always wear grey clothes to participate in any meeting and gathering, at least the bride cannot wear grey clothes at the wedding.

Example 2: In Chinese ancient story of the three kingdoms, as Zhuge Liang command the war, he generally applies The Golden Mean "combining punishment with leniency". The most obvious example is that, in the battle of Red Cliff, he firstly associates with Zhou Yu to beat the army of Cao Cao, and obtains a brilliant victory; but he deliberately sends Guan Yu to ambush at Huarong Road, due to gratitude for the old kindness, Guan Yu and his army loose the powerful enemy of Cao Cao. However, in some cases, Zhuge Liang cannot carry on The Golden Mean. For example, as Ma Su is defeated and losing a place of strategic importance, Zhuge Liang puts him to death without mercy. In addition, Zhuge Liang captures Meng Huo seventh times, and releases him seventh times; it is so tolerant, as rarely seen in history.

Example 3: Some scholars believe that the theoretical foundation of universe is the unity of heaven and man. An instance is as it follows: a boat is travelling from the midstream to the downstream of a river. In Song dynasty, the famous poet Su Dongpo was rafting with guests beneath Red Cliff, and did write the eternal masterpiece "Chibi Fu". For this reason, the men who clings to "The Golden Mean" intends to follow Su Dongpo and write a masterpiece again. Although thousands of writers visit Red Cliff, no one can write a decent poem.

However, according to the viewpoint of "Neutrosophic tetrad" (thesis-antithesis-neutrothesis-neutrosynthesis), one can also boat against the current, sail in the sea, sing in the loess plateau, and the like, in order to write a decent poem.

In short, at the right time and the right place, and having a good authoring environment (similar to what happened when Su Dongpo wrote "Chibi Fu"), the writers can apply different ways to write excellent poetry or other literary works. For example, the "Four Classics" ("A Dream of Red Mansions", "Journey to the West", "The Three Kingdoms", and "Water Margin") were not written by sticking to the stereotypes of Su Dongpo.

Due to space limitations, we no longer discuss other examples and results of the interpretation and expansion of "The Golden Mean".

Conclusion

The "mean", the "middle", or the "neutralities" are neither fixed points; nor rigid rules. The "mean" is not always located at equidistant midpoint between the two opposing sides, and is not always fixed at some point or within a certain range, but it changes with peculiarities like a specific time, a specific location, or a specific condition.

The essences of Chinese traditional culture, including here "The Golden Mean", should adapt with the times, expanding and developing towards "modernization" and "globalization" through international contemporary popular ideas and methods. Applying "Neutrosophic tetrad" (thesis-antithesis-neutrothesis-neutrosynthesis) to re-interpret The Golden Mean is an initial attempt, and we hope to play a significant role, and give a new philosophical direction.

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Hypercomplex Neutrosophic Similarity Measure & Its Application in Multicriteria Decision Making Problem

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Abstract. Neutrosophic set is very useful to express uncertainty, impreciseness, incompleteness and inconsistency in a more general way. It is prevalent in real life application problems to express both indeterminate and inconsistent information. This paper focuses on introducing a new similarity measure in the neutrosophic environment. Similarity measure approach can be used in ranking the alternatives and determining the best among them. It is useful to find the optimum alternative for multi criteria decision making (MCDM) problems from similar alternatives in neutrosophic form. We define a function based on hypercomplex number system in this paper to determine the degree of similarity between single valued neutrosophic sets and thus a new approach to rank the alternatives in MCDM problems has been introduced. The approach of using hypercomplex number system in formulating the similarity measure in neutrosophic set is new and is not available in literature so far. Finally, a numerical example demonstrates how this function determines the degree of similarity between single valued neutrosohic sets and thereby solves the MCDM problem.

Keywords: Hypercomplex similarity measure, Neutrosophic fuzzy set, Decision Making.

1 Introduction

Zadeh introduced the degree of membership/truth (t) in 1965 and defined the fuzzy set which is an extension of ordinary or crisp set as the elements in the fuzzy set are characterised by the grade of membership to the set. Atanassov introduced the degree of nonmembership/falsehood (f) in 1986 and defined the intuitionistic fuzzy set. An intutionistic fuzzy set is characterized by a membership and nonmembership function and thus can be thought of as the extension of fuzzy set. Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set [1]. He has coined the words "neutrosophy" and "neutrosophic". In 2013 he refined the neutrosophic set to n components: $t_1, t_2, \ldots t_i; i_1, i_2, \ldots, i_k; f_1, f_2, \ldots, f_l$, with j+k+l = n > 3. A neutrosophic set generalizes the concepts of classical set, fuzzy set and intutionistic fuzzy set by considering truth-membership function, indeterminacy membership function and falsity-membership function. Real life problems generally deal with indeterminacy, inconsistency and incomplete information which can be best represented by a neutrosophic set.

Properties of neutrosophic sets, their operations, similarity measure between them and solution of MCDM problems in neutrosophic environment are available in the literature. In [2] Wang et al. presented single valued neutrosophic set (SVNS) and defined the notion of inclusion, complement, union, intersection and discussed various properties of set-theoretic operators. They also provided in [3] the set-theoretic operators and various properties of interval valued neutrosophic sets (IVNSs). Said Broumi and Florentin Smarandache introduced the concept of several similarity measures of neutrosophic sets [4]. In this paper they presented the extended Hausdorff distance for neutrosophic sets and defined a series of similarity measures to calculate the similarity between neutrosophic sets. In [5] Ye introduced the concept of a simplified neutrosophic set (SNS), which is a subclass of a neutrosophic set and includes the concepts of IVNS and SVNS; he defined some operational laws of SNSs and proposed simplified neutrosophic weighted averaging (SNWA) operator and simplified neutrosophic weighted geometric (SNWG) operator and applied them to multi criteria decision-making problems under the simplified neutrosophic environment. Ye [6] further generalized the Jaccard, Dice and cosine similarity measures between two vectors in SNSs. Then he applied the three similarity measures to a multi criteria decision-making problem in the simplified neutrosophic setting. Broumi and Smarandache [7] defined weighted interval valued neutrosophic sets and found a cosine similarity measure between two IVNSs. Then they applied it to problems related to pattern recognition.

Various comparison methods are used for ranking the alternatives. Till date no similarity measure using hypercomplex number system in neutrosophic environment is

available in literature. We introduce hypercomplex number in similarity measure. In this paper SVNS is represented as a hypercomplex number. The distance measured between so transformed hypercomplex numbers can give the similarity value. We have used hypercomplex numbers as discussed by Silviu Olariu in [8]. Multiplication of such hypercomplex numbers is associative and commutative. Exponential and trigonometric form exist, also the concept of analytic function, contour integration and residue is defined. Many of the properties of two dimensional complex functions can be extended to hypercomplex numbers in n dimensions and can be used in similarity measure problems. Here in lies the robustness of this method being another application of complex analysis.

The rest of paper is structured as follows. Section 2 introduces some concepts of neutrosophic sets and SNSs. Section 3 describes Jaccard, Dice and cosine similarity measures. In section 4 three dimensional hypercomplex number system and its properties have been discussed. We define a new function based on three dimensional hypercomplex number system for similarity measure to compare neutrosophic sets in section 5. Section 6 demonstrates application of hypercomplex similarity measures in Decision-Making problem. In section 7, a numerical example demonstrates the application and effectiveness of the proposed similarity measure in decision-making problems in neutrosophic environment. We conclude the paper in section 8.

2 Neutrosophic sets

2.1 Definition

Let U be an universe of discourse, then the neutrosophic set A is defined as $A = \{< x: \mathrm{T_A(x)}, \mathrm{I_A(x)}, \mathrm{F_A(x)} >\}$, where the functions $\mathrm{T, I, F: U} \to {]^{-}0, 1^{+}[}$ define respectively the degree of membership (or Truth), the degree of indeterminacy and the degree of non-membership (or falsehood) of the element $x \in U$ to the set A with the condition ${}^{-}0 \leq \mathrm{T_A(x)} + \mathrm{I_A(x)} + \mathrm{F_A(x)} \leq 3^{+}$.

To apply neutrosophic set to science and technology, we consider the neutrosohic set which takes the value from the subset of $[0, 1]$ instead of ${]^{-}0, 1^{+}[}$ i.e., we consider SNS as defined by Ye in [5].

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2.2 Simplified Neutrosophic Set

Let X is a space of points (objects) with generic elements in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function$\mathrm{T_A(x)}$, an indeterminacy membership function $\mathrm{I_A(x)}$, and a falsity membership function $\mathrm{F_A(x)}$ if the functions $\mathrm{T_A(x), I_A(x), F_A(x)}$ are singletons subintervals/subsets in the real standard $[0, 1]$, i.e. $\mathrm{T_A(x): X} \to [0, 1], \mathrm{I_A(x): X} \to [0, 1]$, $\mathrm{F_A(x): X} \to [0, 1]$. Then a simplification of the neutrosophic set A is denoted by $A = \{< x: \mathrm{T_A(x)}, \mathrm{I_A(x)}, \mathrm{F_A(x)} >, x \in X\}$.

2.3 Single Valued Neutrosophic Sets (SVNS)

Let X is a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, for each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. Therefore, a SVNS A can be written as $A_{SVNS} = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$.

For two SVNS, $A_{SVNS} = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\}$ and $B_{SVNS} = \{< x: T_B(x), I_B(x), F_B(x) >, x \in X\}$, the following expressions are defined in [2] as follows:
$A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)$. $A_{NS} = B_{Ns}$ if and only if $T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$. $A^c =< x, F_A(x), 1 - I_A(x), T_A(x) >$

For convenience, a SVNS A is denoted by $A =< T_A(x), I_A(x), F_A(x) >$ for any x$\in X$; for two SVNSs A and B; the operational relations are defined by [2],
$(1) A \cup B =$
$< \max\big(T_A(x), T_B(x)\big), min\big(I_A(x), I_B(x)\big), min\big(T_A(x), T_B(x)\big) >$
$(2) A \cap B =<$
$min\big(T_A(x), T_B(x)\big), max\big(I_A(x), I_B(x)\big), max\big(T_A(x), T_B(x)\big) >$

3 Jaccard, Dice and cosine similarity

The vector similarity measure is one of the most important techniques to measure the similarity between objects. In the following, the Jaccard, Dice and cosine similarity measures between two vectors are introduced

Let $X = (x_1, x_2, \dots, x_n)$ and $Y = (y_1, y_2, \dots, y_n)$ be the two vectors of length n where all the coordinates are positive. The Jaccard index of these two vectors is defined as

$$J(X, Y) = \frac{X.Y}{\|X\|_2{}^2 + \|Y\|_2{}^2 + X.Y} = \frac{\sum_{i=1}^{n} x_i.y_i}{\sum_{i=1}^{n} x_i{}^2 + \sum_{i=1}^{n} y_i{}^2 - \sum_{i=1}^{n} x_i.y_i},$$

where $X.Y = \sum_{i=1}^{n} x_i.y_i$ is the inner product of the vectors X and Y.

The Dice similarity measure is defined as

$$J(X, Y) = \frac{2X.Y}{\|X\|_2{}^2 + \|Y\|_2{}^2} = \frac{2\sum_{i=1}^{n} x_i.y_i}{\sum_{i=1}^{n} x_i{}^2 + \sum_{i=1}^{n} y_i{}^2}$$

Cosine formula is defined as the inner product of these two vectors divided by the product of their lengths. This is the cosine of the angle between the vectors. The cosine similarity measure is defined as

$$C(X,Y)=\frac{X.Y}{\|X\|_2{}^2.\|Y\|_2{}^2}=\frac{\sum_{i=1}^{n}x_i.y_i}{\sum_{i=1}^{n}x_i{}^2.\sum_{i=1}^{n}y_i{}^2}$$

It is obvious that the Jaccard, Dice and cosine similarity measures satisfy the following properties

$$(P_1)\ 0\le J(X,Y),D(X,Y),C(X,Y)\le 1$$

$$(P_2)\ J(X,Y)=J(Y,X),D(X,Y)=D(Y,X)\text{ and }C(X,Y)=C(Y,X)$$

$$(P_3)\ J(X,Y)=1,\ D(X,Y)=1\text{ and }C(X,Y)=1\text{ if }X=Y$$

i.e., $x_i=y_i(i=1,2,\dots,n)$for every $x_i\in X$ and $y_i\in Y$. Also Jaccard, Dice, cosine weighted similarity measures between two SNSs A and B as discussed in [6] are

$$WJ(A,B)=\sum_{i=1}^{n}w_i\frac{\begin{matrix}T_A(x_i)T_B(x_i)\\+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{matrix}}{\begin{matrix}\left(T_A(x_i)\right)^2+\left(I_A(x_i)\right)^2+\left(F_A(x_i)\right)^2\\+\left(T_B(x_i)\right)^2+\left(T_A(x_i)\right)^2+\left(T_A(x_i)\right)^2+\left(T_A(x_i)\right)^2\\-T_A(x_i)T_B(x_i)-T_B(x_i)T_C(x_i)-T_C(x_i)T_A(x_i)\end{matrix}}$$

$$WD(A,B)=\sum_{i=1}^{n}w_i\frac{2\begin{pmatrix}T_A(x_i)T_B(x_i)\\+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{pmatrix}}{\begin{matrix}\left(T_A(x_i)\right)^2+\left(I_A(x_i)\right)^2+\left(F_A(x_i)\right)^2\\+\left(T_B(x_i)\right)^2+\left(T_A(x_i)\right)^2+\left(T_A(x_i)\right)^2+\left(T_A(x_i)\right)^2\end{matrix}}$$

$$WC(A,B)=\sum_{i=1}^{n}w_i\frac{\begin{pmatrix}T_A(x_i)T_B(x_i)\\+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{pmatrix}}{\begin{matrix}\sqrt{\left(T_A(x_i)\right)^2+\left(I_A(x_i)\right)^2+\left(F_A(x_i)\right)^2}\\\sqrt{\left(T_B(x_i)\right)^2+\left(T_A(x_i)\right)^2+\left(T_A(x_i)\right)^2+\left(T_A(x_i)\right)^2}\end{matrix}}$$

4 Geometric representation of hypercomplex number in three dimensions

A system of hypercomplex numbers in three dimensions is described here, for which the multiplication is associative and commutative, which have exponential and trigonometric forms and for which the concepts of analytic tricomplex function, contour integration and residue is defined. The tricomplex numbers introduced here have the form $u=x+hy+kz$, the variables x, y and z being real numbers. The multiplication rules for the complex units h,k are $h^2=k,k^2=h,1.h=h,1.k=k,hk=1$ as discussed in [8]. In a geometric representation, the tricomplex number u is represented by the point P of nates (x,y,z). If O is the origin of the x,y,z axes, (t) the trisector line $x=y=z$ of the positive octant and Π the plane $x+y+z=0$ passing through the origin (O) and perpendicular to (t), then the tricomplex number u can be described by the projection S of the segment OP along the line (t), by the distance D from P to the line (t), and by the azimuthal angle ϕ in the Π plane. It is the angle between the projection of P on the plane Π and the straight line which is the intersection of the plane Π and the plane determined by line t and x axis, $0\le\phi\le 2\pi$. The amplitude ρ of a tricomplex number is defined as $\rho=(x^3+y^3+z^3-3xyz)^{1/3}$, the polar angle θ of OP with respect to the trisector line (t) is given by $tan\,\theta=\frac{D}{S},0\le\theta\le\pi$ and the distance from P to the origin is $d^2=x^2+y^2+z^2$. the tricomplex number $x+hy+kz$ can be represented by the point P of coordinates (x, y, z). The projection S = OQ of the line OP on the trisector line $x=y=z$, which has the unit tangent$\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$, $is\ S=\frac{1}{\sqrt{3}}(x+y+z)$. The distance D = PQ from P to the trisector line $x=y=z$, calculated as the distance from the point P(x, y ,z) to the point Q of coordinates $\left[\frac{x+y+z}{3},\frac{x+y+z}{3},\frac{x+y+z}{3}\right]$, is $D^2=\frac{2}{3}(x^2+y^2+z^2-xy-yz-zx)$. The quantities S and D are shown in Fig. 1, where the plane through the point P and perpendicular to the trisector line (t) intersects the x axis at point A of coordinates $(x+y+z,0,0)$, the y axis at point B of coordinates $(0.x+y+z,0)$, and the z axis at point C of coordinates $(0,0,x+y+z)$. The expression of ϕ in terms of x, y, z can be obtained in a system of coordinates defined by the unit vectors $\xi_1=\frac{1}{\sqrt{6}}(2,-1,-1)$, $\xi_2=\frac{1}{\sqrt{2}}(0,-1,-1),\xi_3=\frac{1}{\sqrt{3}}(1,1,1)$ and having the point O as origin. The relation between the coordinates of P in the systems ξ_1,ξ_2,ξ_3 and x, y, z can be written in the form:

$$\begin{bmatrix}\xi_1\\\xi_2\\\xi_3\end{bmatrix}=\begin{bmatrix}\frac{2}{\sqrt{6}}&-\frac{1}{\sqrt{6}}&-\frac{1}{\sqrt{6}}\\0&-\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{3}}\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$(\xi_1,\xi_2,\xi_3)=\left(\frac{1}{\sqrt{6}}(2x-y-z),\frac{1}{\sqrt{2}}(y-z),\frac{1}{\sqrt{3}}(x+y+z)\right).\ \text{Also}\cos\phi=\frac{2x-y-z}{2\sqrt{(x^2+y^2+z^2-xy-yz-zx)}}$$

$$\sin\phi = \frac{\sqrt{3}(y-z)}{2\sqrt{(x^2+y^2+z^2-xy-yz-zx)}}$$

The angle θ between the line OP and the trisector line (t) is given by $\tan\theta = \frac{D}{S}$

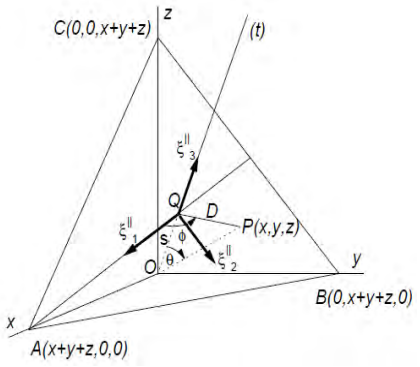

Figure 1: Tricomplex variables s, d, θ, ϕ for the tricomlex number $x + hy + kz$, represented by the point P(x, y, z). The azimuthal angle ϕ is shown in the plane parallel to Π, passing through P, which intersects the trisector line (t) at Q and the axis of coordinates x, y, z at the points A, B, C. The orthogonal axis: $\xi_1^{||}, \xi_2^{||}, \xi_3^{||}$ have the origin at Q. The axis $Q\xi_1^{||}$ is parallel to the axis $O\xi_1^{||}$, the axis $Q\xi_2^{||}$ is parallel to the axis $O\xi_2^{||}$, and the axis $Q\xi_3^{||}$ is parallel to the axis $O\xi_3^{||}$, so that, in the plane ABC, the angle ϕ is measured from the line QA.

5 Hypercomplex similarity measure for SVNS

We here define a function for similarity measure between SVNSs. It requires satisfying some properties of complex number in three dimensions to satisfy the prerequisites of a similarity measure method. In this sense, we can call the function to be defined in three dimensional complex number system or hypercomplex similarity measurement function.

Definition I: Let $A = \{x, T_A(x), I_A(x), F_A(x)\}$ and $B = \{x, T_B(x), I_B(x), F_B(x)\}$ are two neutrosophic sets in $X = \{x\}$; then the similarity function between two neutrosophic sets A and B is defined as

$$S(A,B) = \frac{1}{2}\left[\frac{(1+D_{\theta_1}D_{\theta_2})^2}{1+{D_{\theta_1}}^2+{D_{\theta_2}}^2+{D_{\theta_1}}^2{D_{\theta_2}}^2} + \frac{(1+D_{\phi_1}D_{\phi_2})^2}{1+{D_{\phi_1}}^2+{D_{\phi_2}}^2+{D_{\phi_1}}^2{D_{\phi_2}}^2}\right],$$ where

$$D_{\theta_1} = \frac{\sqrt{(T_A(x)-I_A(x))^2+(I_A(x)-F_A(x))^2+(F_A(x)-T_A(x))^2}}{(T_A(X)+I_A(x)+F_A(x))}$$

$$D_{\theta_2} = \frac{\sqrt{(T_B(x)-I_B(x))^2+(I_B(x)-F_B(x))^2+(F_B(x)-T_B(x))^2}}{(T_B(X)+I_B(x)+F_B(x))}$$

$$D_{\phi_1} = \frac{\sqrt{3}(I_A(x)-F_A(x))}{2T_A(x)-I_A(x)-F_A(x)}$$

$$D_{\phi_2} = \frac{\sqrt{3}(I_B(x)-F_B(x))}{2T_B(x)-I_B(x)-F_B(x)}$$

Also $(T_A(x), I_A(x), F_A(x) \neq (0, 0, 0)$ and $(T_B(x), I_B(x), F_B(x)) \neq (0, 0, 0)$.

Lemma I: Function S (A, B) satisfies the properties of similarity measure.

Proof: Let us consider $S_1(A,B) = \frac{1}{2}\left[\frac{1}{1+\tan^2(\theta_1-\theta_2)} + \frac{1}{1+\tan^2(\varphi_1-\varphi_2)}\right] = \frac{1}{2}\left[\frac{(1+\tan\theta_1\tan\theta_2)^2}{1+\tan^2\theta_1+\tan^2\theta_2+\tan^2\theta_1\tan^2\theta_2} + \frac{(1+\tan\varphi_1\tan\varphi_2)^2}{1+\tan^2\varphi_1+\tan^2\varphi_2+\tan^2\varphi_1\tan^2\varphi_2}\right]$. From (1), (2) and (3), we get the value of $\tan\theta_1, \tan\theta_2, \tan\varphi_1, \tan\varphi_2$. If we take $\tan\theta_1 = D_{\theta_1}$, $\tan\theta_2 = D_{\theta_2}$, $\tan\varphi_1 = D_{\varphi_1}$, $\tan\varphi_2 = D_{\varphi_2}$, then $S_1(A,B) = S(A,B)$

Clearly the function $S_1(A,B)$ satisfies the properties

$(p_1)\ 0 \leq S_1(A,B) \leq 1$

$(P_2)\ S_1(A,B) = S_1(B,A)$

$(P_3)\ When\ A = B,\ \theta_1 = \theta_2\ and\ \phi_1 = \phi_2,\ i.e., S_1(A,B) = 1, if A = B.$

6 Application of Hypercomplex similarity Measures in Decision-Making

In this section, we apply hypercomplex similarity measures between SVNSs to the multicriteria decision-making problem. Let $A = A_1, A_2, \ldots, A_m$ be a set of alternatives and $C = C_1, C_2, \ldots, C_n$ *be a set of criteria.* Assume that the weight of the criterion $C_j (j = 1,2, \ldots, n)$ entered by the decision-maker is w_j, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. The m options according to the n criterion are given below:

	C_1	C_2	C_3	...	C_n
A_1	${C_1}^{(A_1)}$	${C_2}^{(A_1)}$	${C_1}^{(A_1)}$	...	${C_n}^{(A_1)}$
A_2	${C_1}^{(A_2)}$	${C_2}^{(A_2)}$	${C_3}^{(A_2)}$	...	${C_n}^{(A_3)}$
A_3	${C_1}^{(A_3)}$	${C_2}^{(A_3)}$	${C_3}^{(A_3)}$	...	${C_n}^{(A_3)}$
:	:	:	:	:	:
A_m	${C_1}^{(A_n)}$	${C_2}^{(A_3)}$	${C_3}^{(A_3)}$	...	${C_n}^{(A_m)}$

Generally, the evaluation criteria can be categorized into two types: benefit criteria and cost criteria. Let K be a set of benefit criteria and M be a set of cost criteria. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative

$$A^* = \{C_1^*, C_2^*, C_3^*, \dots, C_n^*\}$$

Where for a benefit criterion

$C_j^* = \{max_i T_{C_j}^{(A_i)}, min_i I_{C_j}^{(A_i)}, min_i F_{C_j}^{(A_i)}\}$ while for a cost criterion,

$C_j^* = \{min_i T_{C_j}^{(A_i)}, max_i I_{C_j}^{(A_i)}, max_i F_{C_j}^{(A_i)}\}$

Definition II: We define hypercomplex weighted similarity measure as

$WS_K(A_i, A^*) = \sum_{j=1}^{n} W_j S\left(C_j^{(A_i)}, C_j^*\right), (i = 1, 2, 3, \dots, m)$ Lemma II: $WS_K(A_i, A^*), (i = 1, 2, 3, \dots, m)$ satisfies properties P_1, P_2, P_3.

Proof: Clearly $\sum_{j=1}^{n} w_j S\left(C_j^{(A_i)}, C_j^*\right) \geq 0$ and since from the property of hypercomplex similarity measure $S\left(C_j^{(A_i)}, C_j^*\right) \leq 1$, $\sum_{j=1}^{n} w_j S\left(C_j^{(A_i)}, C_j^*\right) \leq \sum_{j=1}^{n} w_j = 1$, so $0 \leq WS_K(A_i, A^*) \leq 1$. Thus P_1 is satisfied.

Since $S\left(C_j^{(A_i)}, C_j^*\right) = S\left(C_j^*, C_j^{(A_i)}\right), WS_K(A_i, A^*) = WS_K(A^*, A_i)$. Thus P_2 is satisfied.

When $C_j^{(A_i)} = C_j^*$, Using the property of hypercomplex similarity measure

$S\left(C_j^{(A_i)}, C_j^*\right) = 1$, So $\sum_{j=1}^{n} w_j S\left(C_j^{(A_i)}, C_j^*\right) = \sum_{j=1}^{n} w_j = 1$ if $C_j^{(A_i)} = C_j^*$.

So P_3 is also satisfied.

Through the similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected.

7 Numerical Example

In a certain network, there are four options to go from one node to the other. Which path to be followed will be impacted by two benefit criteria C_1, C_2 and one cost criteria C_3 and the weight vectors are 0.35, 0.25 and 0.40 respectively. A decision maker evaluates the four options according to the three criteria mentioned above. We use the newly introduced approach to obtain the most desirable alternative from the decision matrix given in table 1.

C_1, C_2 are benefit criteria, C_3 is cost criteria. From table 1 we can obtain the following ideal alternative:

$A^* = \{(0.7, 0, 0.1), (0.6, 0.1, 0.2), (0.5, 0.3, 0.8)\}$

	A_1	A_2	A_3	A_4
C_1	$(0.4, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$	$(0.3, 0.2, 0.3)$	$(0.7, 0, 0.1)$
C_2	$(0.4, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$	$(0.5, 0.2, 0.3)$	$(0.6, 0.1, 0.2)$
C_3	$(0.8, 0.2, 0.5)$	$(0.5, 0.2, 0.8)$	$(0.5, 0.3, 0.8)$	$(0.6, 0.3, 0.8)$

Table 1: Decision matrix (information given by DM)

Measure method	*measure value*	*Ranking order*
Weighted Jaccarad similarity measure	$\boldsymbol{WJ(A_1, A^*) = 0.7642}$ $\boldsymbol{WJ(A_2, A^*) = 0.9735}$ $\boldsymbol{WJ(A_3, A^*) = 0.8067}$ $\boldsymbol{WJ(A_1, A^*) = 0.9962}$	$\boldsymbol{A_4 > A_2 > A_3 > A_1}$
Weighted Jaccarad similarity measure	$\boldsymbol{WD(A_1, A^*) = 0.8635}$ $\boldsymbol{WD(A_2, A^*) = 0.9864}$ $\boldsymbol{WD(A_3, A^*) = 0.8738}$ $\boldsymbol{WD(A_4, A^*) = 0.9981}$	$\boldsymbol{A_4 > A_2 > A_3 > A_1}$
Weighted cosine similarity measure	$\boldsymbol{WD(A_1, A^*) = 0.8773}$ $\boldsymbol{WD(A_2, A^*) = 0.9882}$ $\boldsymbol{WD(A_3, A^*) = 0.8939}$ $\boldsymbol{WD(A_4, A^*) = 0.9986}$	$\boldsymbol{A_4 > A_2 > A_3 > A_1}$
Weighted hypercomplex similarity measure	$\boldsymbol{W_k S(A_1, A^*) = 0.7211}$ $\boldsymbol{W_k S(A_2, A^*) = 0.9857}$ $\boldsymbol{W_k S(A_3, A^*) = 0.8090}$ $\boldsymbol{W_k S(A_4, A^*) = 0.9895}$	$\boldsymbol{A_4 > A_2 > A_3 > A_1}$

7.1 Generalization of hypercomplex similarity measure

In this section 7, we formulate a general function for similarity measure using hypercomplex number system. This can give similarity measure for any dimension. Before formulating it, we should have a fare knowledge of hypercomplex number in n-dimensions [8] for which the multiplication is associative and commutative, and also the concepts of analytic n-complex function, contour integration and residue is defined. The n-complex number $x_0 + h_1 x_1 + h_2 x_2 + \cdots + h_{n-1} x_{n-1}$ can be represented by the point A of coordinates $(x_0, x_1, \dots, x_{n-1})$ where$h_1, h_2, \dots, h_{n-1}$ are the hypercomplex bases for which the multiplication rules are$h_j h_k = h_{j+k}$ if $0 \leq j + k \leq n - 1, and\ h_j h_k = h_{j+k-n}$ if $n \leq j + k \leq 2n - 2$, where $h_0 = 1$. If O is the origin of the n dimensional space, the distance from the origin O to the point A of coordinates $(x_0, x_1, \dots, x_{n-1})$ has the expression $d^2 = {x_0}^2 + {x_1}^2 + {x_2}^2 + \cdots + {x_{n-1}}^2$. The quantity d will be called modulus of the n-complex number $u = x_0 + h_1 x_1 + h_2 x_2 + \cdots + h_{n-1} x_{n-1}$. The modulus of an n-complex number u will be designated by $d = |u|$. For even number of dimensions $(n \geq 4)$ hypercomplex number is charac-

terized by two polar axis, one polar axis is the normal through the origin O to the hyperplane $v_+ = 0$ where$v_+ = x_0 + x_1 + \cdots + x_{n-1}$ and the second polar axis is the normal through the origin O to the hyperplane $v_- = 0$ where $v_- = x_0 - x_1 + \cdots + x_{n-2} - x_{n-1}$. Whereas for an odd number of dimensions, n-complex number is of one polar axis, normal through the origin O to the hyperplane $v_+ = 0$.

Thus, in addition to the distance d, the position of the point A can be specified, in an even number of dimensions, by two polar angles θ_+, θ_-, by n/2-2 planar angles ψ_k, and by $\frac{n}{2} - 1$ azimuthal angles ϕ_k. In an odd number of dimensions, the position of the point A is specified by d, by one polar angle θ_+, by planar angles $\psi_{k,}$ and by $\frac{n-1}{2}$ azimuthal angles $\phi_{k.}$ The exponential and trigonometric forms of the n-complex number u can be obtained conveniently in a rotated system of axes defined by a transformation

Which, for even n,

$$\begin{bmatrix} \xi_+ \\ \xi_- \\ \vdots \\ \xi_k \\ \eta_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} \\ \frac{1}{\sqrt{n}} & -\frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} & -\frac{1}{\sqrt{n}} \\ \vdots & \vdots & & \vdots & \vdots \\ \sqrt{\frac{2}{n}} & \sqrt{\frac{2}{n}}\cos\frac{2\pi\kappa}{n} & \cdots & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-2)k}{n} & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-1)k}{n} \\ 0 & \sqrt{\frac{2}{n}}\sin\frac{2\pi k}{n} & \cdots & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-2)k}{n} & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-1)k}{n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_0 \\ x_{1:} \\ \vdots \\ \vdots \\ x_{n-1} \end{bmatrix}$$

Here $k = 1,2, \dots, \frac{n}{2} - 1$.

And for odd n,

$$\begin{bmatrix} \xi_+ \\ \xi_- \\ \vdots \\ \xi_k \\ \eta_k \\ \vdots \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{n}} & \frac{1}{\sqrt{n}} & \cdots & \frac{1}{\sqrt{n}} \\ \frac{2}{\sqrt{n}} & \sqrt{\frac{2}{n}}\cos\frac{2\pi}{n} & \cdots & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-1)}{n} \\ 0 & \sqrt{\frac{2}{n}}\sin\frac{2\pi}{n} & \cdots & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-1)}{n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{2}{\sqrt{n}} & \sqrt{\frac{2}{n}}\cos\frac{2\pi k}{n} & \cdots & \sqrt{\frac{2}{n}}\cos\frac{2\pi(n-1)k}{n} \\ 0 & \sqrt{\frac{2}{n}}\sin\frac{2\pi k}{n} & \cdots & \sqrt{\frac{2}{n}}\sin\frac{2\pi(n-1)k}{n} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x_0 \\ x_{1:} \\ \vdots \\ \vdots \\ x_{n-1} \end{bmatrix}$$

Here $k = 1,2, \dots, \frac{n-1}{2}$

Definition III: Let $A = (x_0, x_1, x_2, \dots, x_{n-1})$ and $B = (y_0, y_1, y_2, \dots, y_{n-1})$ be two n dimensional complex numbers. Then similarity measure between A and B is defined as, when n is odd

$$S(A,B) = \frac{1}{n-1}\left[\frac{1}{1+\tan^2\left({\theta_+}^{(A)} - {\theta_+}^{(B)}\right)} + \sum_{k=1}^{\frac{n-1}{2}} \frac{1}{1+\tan^2\left({\phi_k}^{(A)} - {\phi_k}^{(B)}\right)} + \sum_{k=2}^{\frac{n-1}{2}} \frac{1}{1+\tan^2\left({\psi_{k-1}}^{(A)} - {\psi_{k-1}}^{(B)}\right)}\right]$$

And when n is even,

$$S(A,B) = \frac{1}{n-1}\left[\frac{1}{1+\tan^2\left({\theta_+}^{(A)} - {\theta_+}^{(B)}\right)} + \frac{1}{1+\tan^2\left({\theta_-}^{(A)} - {\theta_-}^{(B)}\right)} + \sum_{k=1}^{\frac{n}{2}-1} \frac{1}{1+\tan^2\left({\phi_k}^{(A)} - {\phi_k}^{(B)}\right)} + \sum_{k=2}^{\frac{n}{2}-1} \frac{1}{1+\tan^2\left({\psi_{k-1}}^{(A)} - {\psi_{k-1}}^{(B)}\right)}\right]$$

Here $\tan\theta_+ = \frac{\sqrt{2}\rho_1}{v_+}, \tan\theta_- = \frac{\sqrt{2}\rho_1}{v_-}, \cos\phi_k = \frac{v_k}{\rho_k}, \sin\phi_k = \frac{\check{v}_k}{\rho_k}$, ${\rho_k}^2 = {v_k}^2 + {\check{v}_k}^2$, $v_+ = \sqrt{n}\xi_+$, $v_- = \sqrt{n}\xi_{-,}$ $v_k = \sqrt{\frac{n}{2}}\ \xi_{k,}$ $\check{v}_k = \sqrt{\frac{n}{2}}\ \eta_k$, $\tan\psi_{k-1} = \frac{\rho_1}{\rho_k}$, and also $0 \le \theta_+ \le \pi, 0 \le \theta_- \le \pi, 0 \le \varphi_k \le 2\pi$ and $0 \le \xi_k \le \frac{\pi}{2}$.

It is very clear that $S(\mathrm{A}, \mathrm{B})$ satisfies the three properties of similarity measure.

8 Conclusion

In this paper we first introduced a new method of similarity measure between single valued neutrosohpic sets using hypercomplex number. We set up an example of decision making problem which requires finalizing an optimal path based on some certain criteria. We compared the result of our introduced similarity measure with those of other methods. We can conclude that we can efficiently apply the introduced similarity measure approach in decision making problems and any other similarity measure problems. Later, we proposed a general function for similarity measure.

The proposed similarity measure is based on the concept of hypercomplex number. We can relate the similarity measure with hypercomplex number system. Thus, it opens a new domain of research in finding the solutions of decision making problems related to the network problems by the use of similarity measures based on hypercomplex number system.

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Interval Neutrosophic Multi-Attribute Decision-Making Based on Grey Relational Analysis

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Abstract. The purpose of this paper is to introduce multi-attribute decision making based on the concept of interval neutrosophic sets. While the concept of neutrosophic sets is a powerful tool to deal with indeterminate and inconsistent data, the interval neutrosophic set is also a powerful mathematical tool as well as more flexible to deal with incompleteness. The rating of all alternatives is expressed in terms of interval neutrosophic values characterized by interval truth-membership degree, interval indeterminacy-membership degree, and interval falsity-membership degree. Weight of each attribute is partially known to the decision maker. The authors have extended the single valued neutrosophic grey relational analysis method to interval neutrosophic environment and applied it to multi-attribute decision making problem. Information entropy method is used to obtain the unknown attribute weights. Accumulated arithmetic operator is defined to transform interval neutrosophic set into single value neutrosophic set. Neutrosophic grey relational coefficient is determined by using Hamming distance between each alternative to ideal interval neutrosophic estimates reliability solution and the ideal interval neutrosophic estimates unreliability solution. Then interval neutrosophic relational degree is defined to determine the ranking order of all alternatives. Finally, an example is provided to illustrate the applicability and effectiveness of the proposed approach.

Keywords: Accumulated arithmetic operator**,** Grey relational analysis, Ideal interval neutrosophic estimates reliability solution, Information entropy, Interval neutrosophic set, Multi-attribute decision making, Neutrosophic set, Single-valued neutrosophic set.

1. Introduction

The concept of neutrosophic sets was introduced by Smarandache [1, 2, 3, 4]. The root of neutrosophic set is the neutrosophy, a new branch of philosophy [1]. The thrust of the neutrosophy creates new field of study such as neutrosophic statistics [5], neutrosophic integral [6], neutrosophic cognitive map [7], etc. The concept of neutrosophic set has been successful in penetrating different branches of sciences [8], social sciences [9, 10, 11], education [12], conflict resoltion [13, 14], philosophy [15], artificial intelligence and control systems [16], etc. Neutrosophic set has drawn the great attention of the researchers for its capability of handling uncertainty, indeterminacy and incomplete information.

Zadeh [17] proposed the degree of membership in 1965 and defined the fuzzy set. Atanassov [18] proposed the degree of non-membership in 1986 and defined the intuitionistic fuzzy set. Smarandache [1] proposed the degree of indeterminacy as independent component and defined the neutrosophic set.

To use neutrosophic sets in practical fields such as real scientific and engineering applications, Wang et al.[19] restricted the concept of neutrosophic set to single valued neutrosophic set since single value is an instance of set value. Neutrosophic set and its various extensions have been studied and applied in different fields such as medical diagnosis [20, 21, 22, 23, 24], decision making [25, 26, 27, 28, 29, 30, 31], decision making in hybrid system [32, 33, 34, 35, 36], image processing [37, 38, 39, 40, 41, 42], etc.

However, Zhang et al. [43] opinioned that in many real world problems, the decision information may be suitably presented by interval form instead of real numbers. In order to deal with the situation, Wang et al.[44] introduced the concept of interval neutrosophic set (INS) characterized by a membership function, non-membership function and an indeterminacy function, whose values are interval forms.

Broumi and Smarandache [45] studied correlation coefficient of interval neutrosophic sets and applied it in medical diagnosis. Broumi and Smarandache [46] studied cosine similarity measure in interval neutrosophic environment. Zhang et al. [43] studied interval neutrosophic sets and its application in multi attribute decision making. Ye [47] studied similarity measures between interval neutrosophic sets and their applications in multicriteria decision making. Ye [48] proposed improved correlation coefficient of SVNS and studied some of its

properties, and then extended it to a correlation coefficient between INS. Chi and Liu [49] developed the order performance technique based on similarity to ideal solution (TOPSIS) method for multiple attribute decision making problems based on interval neutrosophic set.

Grey relational analysis (GRA) studied by Deng [50, 51] is widely used for multi-attribute decision making problems.

Zhang et al. 2005 [52] presented GRA method for multi attribute decision-making with interval numbers. Rao & Singh [53] proposed improved GRA method by integrating analytic hierarchy process and fuzzy logic Wei [54] studied the GRA method for intuitionistic fuzzy multi-criteria decision-making. Pramanik and Mukhopadhyaya [55] presented GRA based intuitionistic fuzzy multi-criteria group decision-making approach for teacher selection in higher education. Biswas et al. [56] proposed entropy based GRA method for multi-attribute decision making under single valued neutrosophic assessments. Biswas et al. [57] also studied GRA based neutrosophic multi-attribute decision-making (MADM) with unknown weight information Mondal and Pramanik [58] applied GRA based neutrosophic decision making model of school choice.

GRA based MADM in interval neutrosophic environment is yet to appear in the literature. In this paper, we present interval neutrosophic multi attribute decision making based on GRA.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets and interval neutrosophic sets. Section 3 is devoted to present GRA method for multi attribute decision-making in interval neutrosophic environment. Section 4 presents a numerical example of the proposed method. Finally section 5 presents concluding remarks.

2 Mathematical preliminaries

2.1 Definitions on neutrosophic Set [1]

Definition 2.1.1: Let E be a space of points (objects) with generic element in E denoted by x. Then a neutrosophic set P in E is characterized by a truth membership function $T_P(x)$, an indeterminacy membership function $I_P(x)$ and a falsity membership function $F_P(x)$. The functions $T_P(x)$, $I_P(x)$ and $F_P(x)$ are real standard or non-standard subsets of $]^-0,1^+[$ that is $T_P(x): E \to]^-0,1^+[$; $I_P(x): E \to]^-0,1^+[$; $F_P(x): E \to]^-0,1^+[$.

The sum of $T_P(x)$, $I_P(x)$, $F_P(x)$ satisfies the relation

$$^-0 \le \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \le 3^+$$

Definition 2.1.2 (complement) [1]

The complement of a neutrosophic set P is denoted by P^c and is defined as follows:

$T_{P^c}(x) = \{1^+\} - T_P(x)$; $I_{P^c}(x) = \{1^+\} - I_P(x)$, and $F_{P^c}(x) = \{1^+\} - F_P(x)$

Definition 2.1.3 (Containment) [1]

A neutrosophic set P is contained in the other neutrosophic set Q, $P \subseteq Q$ if and only if the following result holds.

$\inf T_P(x) \le \inf T_Q(x)$, $\sup T_P(x) \le \sup T_Q(x)$

$\inf I_P(x) \ge \inf I_Q(x)$, $\sup I_P(x) \ge \sup I_Q(x)$

$\inf F_P(x) \ge \inf F_Q(x)$, $\sup F_P(x) \ge \sup F_Q(x)$

for all x in E.

Definition 2.1.4 (Single-valued neutrosophic set) [19]

Let E be a universal space of points (objects) with a generic element of E denoted by x.

A single valued neutrosophic set [Wang et al. 2010] S is characterized by a truth membership function $T_S(x)$, a falsity membership function $F_S(x)$ and indeterminacy function $I_S(x)$ with $T_S(x), F_S(x), I_S(x) \in [0,1]$ for all x in E. ,

When E is continuous, a SNVS S can be written as follows:

$$S = \int_x \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in E$$

and when E is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in E$$

It should be observed that for a SVNS S,

$$0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3, \ \forall x \in E$$

Definition 2.1.5: The complement of a single valued neutrosophic set S [19] is denoted by S^c and is defined by

$T_S{}^c(x) = F_S(x)$; $I_S{}^c(x) = 1 - I_S(x)$; $F_S{}^c(x) = T_S(x)$

Definition 2.1.6: A SVNS S_P [19] is contained in the other SVNS S_Q, denoted as $S_P \subseteq S_Q$ iff, $T_{S_P}(x) \le T_{S_Q}(x)$;

$I_{S_P}(x) \ge I_{S_Q}(x)$; $F_{S_P}(x) \ge F_{S_Q}(x)$, $\forall x \in E$.

Definition 2.1.7: Two single valued neutrosophic sets S_P and S_Q [19] are equal, i.e. $S_P = S_Q$, iff, $S_P \subseteq S_Q$ and $S_P \supseteq S_Q$.

Definition 2.1.8: (Union) The union of two SVNSs S_P and S_Q [19] is a SVNS S_R, written as $S_R = S_P \cup S_Q$ .Its truth membership, indeterminacy-membership and falsity membership functions are related to S_P and S_Q by the relations as follows:

$T_{S_R}(x) = \max\left(T_{S_P}(x), T_{S_Q}(x)\right)$;

$I_{S_R}(x) = \max\left(I_{S_P}(x), I_{S_Q}(x)\right)$;

$F_{S_R}(x) = \min\left(F_{S_P}(x), F_{S_Q}(x)\right)$ for all x in E

Definition 2.1.9 (Intersection) [19]

The intersection of two SVNSs P and Q is a SVNS V, written as $V = P \cap Q$. Its truth membership, indeterminacy membership and falsity membership functions are related to P an Q by the relations as follows:

$T_{S_V}(x) = \min\left(T_{S_P}(x), T_{S_Q}(x)\right)$;

$I_{S_V}(x) = \max\left(I_{S_P}(x), I_{S_Q}(x)\right)$;

$F_{S_V}(x) = \max\left(F_{S_P}(x), F_{S_Q}(x)\right), \forall x \in E$

Distance between two neutrosophic sets.

The SVNS can be presented in the following form:

$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in E\}$

Finite SVNSs can be represented as follows:

$$S = \begin{Bmatrix} (x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, \\ (x_m/(T_S(x_m), I_S(x_m), F_S(x_m))) \end{Bmatrix}, \forall x \in E \quad (1)$$

Definition 2.1.10: Let

$$S_P = \begin{Bmatrix} (x_1/(T_{S_P}(x_1), I_{S_P}(x_1), F_{S_P}(x_1))), \cdots, \\ (x_n/(T_{S_P}(x_n), I_{S_P}(x_n), F_{S_P}(x_n))) \end{Bmatrix} \quad (2)$$

$$S_Q = \begin{Bmatrix} (x_1/(T_{S_Q}(x_1), I_{S_Q}(x_1), F_{S_Q}(x_1))), \cdots, \\ (x_n/(T_{S_Q}(x_n), I_{S_Q}(x_n), F_{S_Q}(x_n))) \end{Bmatrix} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance [59] between two SNVS P and Q is defined as follows:

$$d_S(S_P, S_Q) = \sum_{i=1}^{n} \left\langle \begin{array}{l} \left|T_{S_P}(x) - T_{S_Q}(x)\right| + \\ \left|I_{S_P}(x) - I_{S_Q}(x)\right| + \\ \left|F_{S_P}(x) - F_{S_Q}(x)\right| \end{array} \right\rangle \quad (4)$$

and normalized Hamming distance [59] between two SNVS S_P and S_Q is defined as follows:

$$^{N}d_S(S_P, S_Q) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{array}{l} \left|T_{S_P}(x) - T_{S_Q}(x)\right| + \\ \left|I_{S_P}(x) - I_{S_Q}(x)\right| + \\ \left|F_{S_P}(x) - F_{S_Q}(x)\right| \end{array} \right\rangle \quad (5)$$

with the following properties

1. $0 \le d_S(S_P, S_Q) \le 3n$ (6)

2. $0 \le {}^{N}d_S(S_P, S_Q) \le 3n$ (7)

Definition 2.1.11

Let α and β be the collection of benefit attributes and cost attributes, respectively. R_S^+ is the interval relative neutrosophic positive ideal solution (IRNPIS) and R_S^- is the interval relative neutrosophic negative ideal solution (IRNNIS). $R_S^+ = [r_{S_1}^+, r_{S_2}^+, \cdots, r_{S_n}^+]$ is defined as a solution in which every component $r_{S_j}^+ = \langle T_j^+, I_j^+, F_j^+ \rangle$ is characterized by $T_j^+ = \{(\max_i \{T_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\min_i \{T_{ij}\}) \mid j\text{-}th attribute \in \beta\}$

Definition 2.1.12

The interval relative neutrosophic negative ideal solution (IRNNIS) $R_S^- = [r_{S_1}^-, r_{S_2}^-, \cdots, r_{S_n}^-]$ is a solution in which every component $r_{S_j}^- = \langle T_j^-, I_j^-, F_j^- \rangle$ is characterized as follows:

$T_j^- = \{(\min_i \{T_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\max_i \{T_{ij}\}) \mid j\text{-}th attribute \in \beta\}$,

$I_j^- = \{(\max_i \{I_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\min_i \{I_{ij}\}) \mid j\text{-}th attribute \in \beta\}$,

$F_j^- = \{(\max_i \{F_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\min_i \{F_{ij}\}) \mid j\text{-}th attribute \in \beta\}$,

in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$ (see equation 8) for i = 1, 2, …, n and j = 1, 2, …, m

2.2 Interval Neutrosophic Sets

Definition 2.2 [44]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (INS) M in X is characterized by truth-membership function $T_M(x)$, indeterminacy-membership $I_M(x)$, function and falsity-membership function $F_M(x)$. For each point x in X, we have, $T_M(x), I_M(x), F_M(x) \in [0, 1]$.

For two IVNS,

$M_{INS} = \{< x, ([T_M^L(x), T_M^U(x)], [I_M^L(x), I_M^U(x)], [F_M^L(x), F_M^U(x)]) > \mid x \in X\}$

and $N_{INS} = \{< x, ([T_N^L(x), T_N^U(x)], [I_N^L(x), I_N^U(x)], [F_N^L(x), F_N^U(x)]) > \mid x \in X\}$, the two relations are defined as follows:

(1) $M_{INS} \subseteq N_{INS}$ if and only if $T_M^L \le T_N^L$, $T_M^U \le T_N^U$; $I_M^L \le I_N^L$, $F_M^L \le F_N^L$; $F_M^L \le F_N^L$, $F_M^L \le F_N^L$

(2) $M_{INS} = N_{INS}$ if and only if $T_M^L = T_N^L$, $T_M^U = T_N^U$; $I_M^L = I_N^L$, $F_M^L = F_N^L$; $F_M^L = F_N^L$, $F_M^L = F_N^L$ $\forall x \in X$

3. Grey relational analysis method for multi attributes decision-making in interval neutrosophic environment.

Consider a multi-attribute decision making problem with m alternatives and n attributes. Let A_1, A_2, ..., A_m and C_1, C_2, ...,C_n denote the alternatives and attributes respectively. The rating describes the performance of alternative A_i against attribute C_j. Weight vector $W = \{w_1, w_2, ..., w_n\}$ is assigned to the attributes. The weight w_j (j = 1, 2, ..., n) reflects the relative importance of attributes C_j (j = 1, 2, ..., m) to the decision maker. The values associated with the

alternatives for MADM problems presented in the following table.

Table1: Interval neutrosophic decision matrix

$D_s = \langle d_{ij} \rangle_{m\times n} =$

	C_1	C_2	⋯	C_n
A_1	$\langle d_{11} \rangle$	$\langle d_{12} \rangle$	...	$\langle d_{1n} \rangle$
A_2	$\langle d_{12} \rangle$	$\langle d_{22} \rangle$	...	$\langle d_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle d_{m1} \rangle$	$\langle d_{m2} \rangle$	...	$\langle d_{mn} \rangle$

(8)

Here $\langle d_{ij} \rangle$ is the interval neutrosophic number related to the *i*-th alternative and the *j*-th attribute.

Grey relational analysis (GRA) is one of the adoptive methods for MADM. The steps of GRA under interval neutrosophic environments are described below.

Step1: Determination the criteria

There are many attributes in decision making problems. Some of them are important and others may be less important. So it is necessary to select the proper criteria for decision making situations. The most important criteria may be fixed with help of experts' opinions.

Step 2: Data pre-processing and construction of the decision matrix with interval neutrosophic form

It may be mentioned here that the original GRA method can deal mainly with quantitative attributes. There exists some complexity in the case of qualitative attributes. In the case of a qualitative attribute (quantitative value is not available), an assessment value is taken as interval neutrosophic environment.

For multiple attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed as interval neutrosophic sets. It can be represented with the following forms:

$$A_i = \left| \begin{array}{l} C_1 \Big/ \left\langle N_1\left(\left[T_1^L, T_1^U\right], \left[I_1^L, I_1^U\right], \left[F_1^L, F_1^U\right]\right)\right\rangle, \\ C_2 \Big/ \left\langle N_2\left(\left[T_2^L, T_2^U\right], \left[I_2^L, I_2^U\right], \left[F_2^L, F_2^U\right]\right)\right\rangle, \cdots, \\ C_n \Big/ \left\langle N_n\left(\left[T_n^L, T_n^U\right], \left[I_n^L, I_n^U\right], \left[F_n^L, F_n^U\right]\right)\right\rangle : C_j \in C \end{array} \right|$$

$$= \left| C_j \Big/ \left\langle N_j\left(\left[T_j^L, T_j^U\right], \left[I_j^L, I_j^U\right], \left[F_j^L, F_j^U\right]\right)\right\rangle : C_j \in C \right|$$

for j = 1, 2,…, n (9)

Here $\left\langle N_j\left(\left[T_j^L, T_j^U\right], \left[I_j^L, I_j^U\right], \left[F_j^L, F_j^U\right]\right)\right\rangle$, (j = 1, 2, ..., n) is the interval neutrosophic set with the degrees of interval truth membership $\left[T_j^L, T_j^U\right]$, the degrees of interval indeterminacy membership $\left[I_j^L, I_j^U\right]$ and the degrees of interval falsity membership $\left[F_j^L, F_j^U\right]$ of the alternative A_i satisfying the attribute C_j

The interval neutrosophic decision matrix can be represented in the following form (see the Table 2):

Table2: Interval neutrosophic decision matrix

$d_N = \left\langle \left[T_{ij}^L, T_{ij}^U\right], \left[I_{ij}^L, I_{ij}^U\right], \left[F_{ij}^L, F_{ij}^U\right] \right\rangle_{m\times n} =$

	C_1	C_2	...	C_n
A_1	$\left\langle \begin{matrix} \left[T_{11}^L, T_{11}^U\right], \\ \left[I_{11}^L, I_{11}^U\right], \\ \left[F_{11}^L, F_{11}^U\right] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} \left[T_{12}^L, T_{12}^U\right], \\ \left[I_{12}^L, I_{12}^U\right], \\ \left[F_{12}^L, F_{12}^U\right] \end{matrix} \right\rangle$	...	$\left\langle \begin{matrix} \left[T_{1n}^L, T_{1n}^U\right], \\ \left[I_{1n}^L, I_{1n}^U\right], \\ \left[F_{1n}^L, F_{1n}^U\right] \end{matrix} \right\rangle$
A_2	$\left\langle \begin{matrix} \left[T_{21}^L, T_{21}^U\right], \\ \left[I_{21}^L, I_{21}^U\right], \\ \left[F_{21}^L, F_{21}^U\right] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} \left[T_{22}^L, T_{22}^U\right], \\ \left[I_{22}^L, I_{22}^U\right], \\ \left[F_{22}^L, F_{22}^U\right] \end{matrix} \right\rangle$	...	$\left\langle \begin{matrix} \left[T_{2n}^L, T_{2n}^U\right], \\ \left[I_{2n}^L, I_{2n}^U\right], \\ \left[F_{2n}^L, F_{2n}^U\right] \end{matrix} \right\rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\left\langle \begin{matrix} \left[T_{m1}^L, T_{m1}^U\right], \\ \left[I_{m1}^L, I_{m1}^U\right], \\ \left[F_{m1}^L, F_{m1}^U\right] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} \left[T_{m2}^L, T_{m2}^U\right], \\ \left[I_{m2}^L, I_{m2}^U\right], \\ \left[F_{m2}^L, F_{m2}^U\right] \end{matrix} \right\rangle$	...	$\left\langle \begin{matrix} \left[T_{mn}^L, T_{mn}^U\right], \\ \left[I_{mn}^L, I_{mn}^U\right], \\ \left[F_{mn}^L, F_{mn}^U\right] \end{matrix} \right\rangle$

(10)

Step 3: Determination of the accumulated arithmetic operator

Let us consider an interval neutrosophic set as $\left\langle N_j\left(\left[T_j^L, T_j^U\right], \left[I_j^L, I_j^U\right], \left[F_j^L, F_j^U\right]\right)\right\rangle$.

We transform the interval neutrosophic number to SVNSs by the following operator. The accumulated arithmetic operator (AAO) is defined as follows:

$$N_{ij}\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle =$$

$$N_{ij}\left\langle \left[\frac{T_{ij}^L + T_{ij}^U}{2}\right], \left[\frac{I_{ij}^L + I_{ij}^U}{2}\right], \left[\frac{F_{ij}^L + F_{ij}^U}{2}\right] \right\rangle \quad (11)$$

The decision matrix is transformed in the form of SVNSs as follows:

Table3: Single valued neutrosophic decision matrix in transformed form

$d_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m\times n} =$

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11} \rangle$	$\langle T_{12}, I_{12}, F_{12} \rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n} \rangle$
A_2	$\langle T_{21}, I_{21}, F_{21} \rangle$	$\langle T_{22}, I_{22}, F_{22} \rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{m1} \rangle$	$\langle T_{m2}, I_{m2}, F_{m2} \rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn} \rangle$

(12)

Step 4: Determination of the weights of the criteria

During decision-making process, decision maker may often encounter unknown or partial attribute weights. In many cases, the importance of attributes to the decision maker is not equal. So, it is necessary to determine attribute weight for decision making.

3.1 Method of entropy:

Entropy plays an important role for measuring uncertain information. Majumdar and Samanta [59] developed some similarity and entropy measures for SVNSs. The entropy measure can be used to determine the attributes weights when these are unequal and completely unknown to decision maker.

Now, using AAO operator, we transform all interval neutrosophic numbers to single valued neutrosophic numbers. In this paper for entropy measure of an INS, we consider the following notation:

$$T_{S_P}(x_i)=\left[\frac{T_{ij}^L+T_{ij}^U}{2}\right], I_{S_P}(x_i)=\left[\frac{I_{ij}^L+I_{ij}^U}{2}\right],$$

$$F_{S_P}(x_i)=\left[\frac{F_{ij}^L+F_{ij}^U}{2}\right]$$

We write, $S_P=\langle T_{S_P}(x_i), I_{S_P}(x_i), F_{S_P}(x_i)\rangle$. Then, entropy value is defined as follows:

$$E_i(S_P)=$$

$$1-\frac{1}{n}\sum_{i=1}^{m}(T_{S_P}(x_i)+F_{S_P}(x_i))\left|I_{S_P}(x_i)-I^c{}_{S_P}(x_i)\right| \quad (13)$$

Entropy has the following properties:

1. $E_i(S_P)=0 \Rightarrow S_P$ *is a crisp set and* $I_{S_P}(x_i)=0$ and $F_{S_P}(x_i)=0\ \forall x\in E$.
2. $E_i(S_P)=1 \Rightarrow \langle T_{S_P}(x_1), I_{S_P}(x_1), F_{S_P}(x_1)\rangle = \langle T_{S_P}(x_i), 0.5, F_{S_P}(x_i)\rangle \quad \forall x\in E.$
3. $E_i(S_P)\geq E_i(S_Q)\Rightarrow$ $(T_{S_P}(x_i)+F_{S_P}(x_i))\leq(T_{S_Q}(x_i)+F_{S_Q}(x_i))$ *and* $\left|I_{S_P}(x_i)-I^c{}_{S_P}(x_i)\right|\leq\left|I_{S_Q}(x_i)-I^c{}_{S_Q}(x_i)\right|$
4. $E_i(S_P)=E_i(S_{P^c})\ \forall x\in E.$

In order to obtain the entropy value E_j of the j-th attribute C_j (j = 1, 2,…, n), the equation (13) can be written as follows:

$$E_j=1-\frac{1}{n}\sum_{i=1}^{m}(T_{ij}(x_i)+F_{ij}(x_i))\left|I_{ij}(x_i)-I_{ij}^C(x_i)\right|$$

for i = 1, 2, …, m; j = 1, 2, …, n (14)

It is observed that $E_j\in[0,1]$. Due to Hwang and Yoon [60], the entropy weight of the j-th attribute C_j is presented as follows:

$$W_j=\frac{1-E_j}{\sum_{j=1}^{n}(1-E_j)} \quad (15)$$

We have weight vector W = $(w_1, w_2,\ldots,w_n)^T$ of attributes C_j (j = 1, 2, …, n) with $w_j\geq 0$ and $\sum_{i=1}^{n}w_j=1$.

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS) for interval neutrosophic decision matrix

For an interval neutrosophic decision making matrix $D_S=\langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$, T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i satisfying the attribute C_j. The interval neutrosophic estimate reliability solution (see definition 2.1.11, and 2.1.12) can be determined from the concept of SVNS cube [61].

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Grey relational coefficient of each alternative from IINERS is:

$$G_{ij}^{+}=\frac{\min\limits_i\min\limits_j\Delta_{ij}^{+}+\rho\max\limits_i\max\limits_j\Delta_{ij}^{+}}{\Delta_{ij}^{+}+\rho\max\limits_i\max\limits_j\Delta_{ij}^{+}}, \text{ where}$$

$$\Delta_{ij}^{+}=d\left(q_{S_j}^{+}, q_{S_{ij}}\right),\ i=1, 2, \ldots,\text{m and } j=1, 2, \ldots,\text{n} \quad (16)$$

Grey relational coefficient of each alternative from IINEURS is:

$$G_{ij}^{-}=\frac{\min\limits_i\min\limits_j\Delta_{ij}^{-}+\rho\max\limits_i\max\limits_j\Delta_{ij}^{-}}{\Delta_{ij}^{-}+\rho\max\limits_i\max\limits_j\Delta_{ij}^{-}}, \text{ where}$$

$$\Delta_{ij}^{-}=d\left(q_{S_{ij}}, q_{S_{ij}}^{-}\right),\ i=1, 2, \ldots,\text{m and } j=1, 2, \ldots,\text{n} \quad (17)$$

$\rho\in[0,1]$ is the distinguishable coefficient or the identification coefficient. It is used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho=1$, the comparison environment is unchanged. When $\rho=0$, the comparison environment disappears. Smaller value of distinguishing coefficient will reflect the large range of grey relational coefficient. Generally, $\rho=0.5$ is fixed for decision making.

Step 7: Calculation of the interval neutrosophic grey relational coefficient

Calculate the degree of interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS using the following two equations respectively:

$$G_i^{+}=\sum_{j=1}^{n}w_jG_{ij}^{+} \text{ for } i=1, 2, \ldots,\text{m} \quad (18)$$

$$G_i^{-}=\sum_{j=1}^{n}w_jG_{ij}^{-} \text{ for } i=1, 2, \ldots,\text{m} \quad (19)$$

Step 8: Calculation of the interval neutrosophic relative relational degree

Calculate the interval neutrosophic relative relational degree of each alternative from ITFPIS (indeterminacy truthfulness falsity positive ideal solution) with the help of following two equations:

$$R_i = \frac{G_i^+}{G_i^- + G_i^+}, \text{ for i = 1, 2, …,m} \qquad (20)$$

Step 9: Rank the alternatives

The ranking order of alternatives can be determined based on the interval relative relational degree. The highest value of R_i reflects the most desirable alternative.

Step 10: End

4. Illustrative examples

In this section, interval neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach.

4.1 Example 1

Consider a decision-making problem adapted from [58] studied by Mondal and Pramanik. Suppose a legal guardian wants to get his/her child admitted to a suitable school for proper basic education. There is a panel with three possible alternatives (schools) to get admitted his/her child: (1) A_1 is a Christian missionary school; (2) A_2 is a Basic English medium school; (3) A_3 is a Bengali medium kindergarten. The proposed decision making method can be arranged in the following steps.

Step 1: Determination the most important criteria

The legal guardian must take a decision based on the following four criteria: (1) C_1 is the distance and transport; (2) C_2 is the cost; (3) C_3 is the staff and curriculum; and (4) C_4 is the administration and other facilities.

Step 2: Data pre-processing and Construction of the decision matrix with interval neutrosophic form

We obtain the following interval neutrosophic decision matrix based on the experts' assessment:

Table4. Decision matrix with interval neutrosophic number

$$d_S = \left\langle \left[T_{ij}^L, T_{ij}^U\right], \left[I_{ij}^L, I_{ij}^U\right], \left[F_{ij}^L, F_{ij}^U\right] \right\rangle_{3\times 4} =$$

	C_1	C_2	C_3	C_4
A_1	⟨[0.6, 0.8], [0.2, 0.4], [0.3, 0.5]⟩	⟨[0.6, 0.8], [0.2, 0.4], [0.1, 0.3]⟩	⟨[0.6, 0.8], [0.1, 0.3], [0.3, 0.5]⟩	⟨[0.7, 0.9], [0.2, 0.4], [0.2, 0.4]⟩
A_2	⟨[0.5, 0.7], [0.3, 0.5], [0.1, 0.3]⟩	⟨[0.7, 0.9], [0.4, 0.6], [0.3, 0.5]⟩	⟨[0.6, 0.8], [0.2, 0.4], [0.1, 0.3]⟩	⟨[0.7, 0.9], [0.3, 0.5], [0.4, 0.6]⟩
A_3	⟨[0.5, 0.7], [0.2, 0.4], [0.4, 0.6]⟩	⟨[0.6, 0.8], [0.5, 0.7], [0.1, 0.3]⟩	⟨[0.6, 0.8], [0.4, 0.6], [0.4, 0.6]⟩	⟨[0.7, 0.9], [0.3, 0.5], [0.3, 0.5]⟩

(21)

Step 3: Determination of the accumulated arithmetic operator (AAO)

Using accumulated arithmetic operator (AAO) from equation (11) we have the decision matrix in SVNS form is presented as follows:

Table5: single valued neutrosophic decision matrix in transformed form

	C_1	C_2	C_3	C_4
A_1	⟨0.7, 0.3, 0.4⟩	⟨0.7, 0.3, 0.2⟩	⟨0.7, 0.2, 0.4⟩	⟨0.8, 0.3, 0.3⟩
A_2	⟨0.6, 0.4, 0.2⟩	⟨0.8, 0.5, 0.4⟩	⟨0.7, 0.3, 0.2⟩	⟨0.8, 0.4, 0.5⟩
A_3	⟨0.6, 0.3, 0.5⟩	⟨0.7, 0.6, 0.2⟩	⟨0.7, 0.5, 0.5⟩	⟨0.8, 0.4, 0.4⟩

(22)

Step 4: Determination of the weights of the attributes

Entropy value E_j of the j-th (j = 1, 2, 3, 4) attributes can be determined from the decision matrix d_S (21) and equation (14) as: E_1= 0.6533, E_2 = 0.8200, E_3 = 0.6600, E_4 = 0.6867. Then the corresponding entropy weights w_j, (j = 1, 2, 3, 4) of the attribute C_j (j = 1, 2, 3, 4) according to equation (15) is obtained as w_1 = 0.2938, w_2 = 0.1568, w_3 = 0.2836, w_4 = 0.2658 such that $\sum_{j=1}^{4} w_j = 1$

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS)

The ideal interval neutrosophic estimates reliability solution (IINERS) is presented as it follows:

$$Q_S^+ = \left\langle q_{S_1}^+, q_{S_2}^+, q_{S_3}^+, q_{S4}^+ \right\rangle =$$

$$\left[\begin{array}{l} \left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\}\right\rangle, \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\}\right\rangle, \\ \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\}\right\rangle, \left\langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\}\right\rangle \end{array}\right]$$

$$= \left[\langle 0.7, 0.3, 0.2\rangle, \langle 0.8, 0.3, 0.2\rangle, \langle 0.7, 0.2, 0.2\rangle, \langle 0.8, 0.3, 0.3\rangle\right]$$

The ideal interval neutrosophic estimates un-reliability solution (IINEURS) is presented as follows:

$$Q_S^- = \left\langle q_{S_1}^-, q_{S_2}^-, q_{S_3}^-, q_{S4}^- \right\rangle =$$

$$\left[\begin{array}{l} \left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\}\right\rangle, \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\}\right\rangle, \\ \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\}\right\rangle, \left\langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\}\right\rangle \end{array}\right]$$

$$= \left[\langle 0.6, 0.4, 0.5\rangle, \langle 0.7, 0.6, 0.4\rangle, \langle 0.7, 0.5, 0.5\rangle, \langle 0.8, 0.4, 0.5\rangle\right]$$

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Using the equation (16) the interval neutrosophic grey relational coefficient of each alternative from IINERS can be obtained as the following matrix.

$$\left[G_{ij}^{+}\right]_{3\times4}=\begin{bmatrix}0.8000 & 1.0000 & 0.8000 & 0.6667\\ 0.8000 & 0.5714 & 1.0000 & 0.4444\\ 0.5714 & 0.5714 & 0.4444 & 0.5000\end{bmatrix} \quad (23)$$

Similarly, from the equation (17) the interval neutrosophic grey relational coefficient of each alternative from IINEURS is presented as follows:

$$\left[G_{ij}^{-}\right]_{3\times4}=\begin{bmatrix}0.4545 & 0.3333 & 0.3846 & 0.4545\\ 0.4545 & 0.5556 & 0.3333 & 1.0000\\ 0.7143 & 0.5556 & 1.0000 & 0.7143\end{bmatrix} \quad (24)$$

Step 7: Determine the degree of interval neutrosophic grey relational co-efficient of each alternative from IINERS and IINEURS. The required interval neutrosophic grey relational co-efficient corresponding to IINERS is obtained by using the equation (18) as follows:

G_1^{+} = 0.7961, G_2^{+} = 0.7264, G_3^{+} = 0.5164 (25)

and corresponding to IINEURS is obtained with the help of equation (19) as follows:

G_1^{-} = 0.4156, G_2^{-} = 0.5810, G_3^{-} = 0.7704 (26)

Step 8: Thus interval neutrosophic relative degree of each alternative from IINERS can be obtained with the help of equation (20) as follows:

R_1 = 0.6570, R_2 = 0.5556, R_3 = 0.4013 (27)

Step 9: The ranking order of all alternatives can be determined according to the decreasing order of the value of interval neutrosophic relational degree i.e. $R_1>R_2>R_3$. It is seen that the highest value of interval neutrosophic relational degree is R_1 therefore A_1 (Christ missionary school) is the best alternative (school) for his/her the child for getting admission.

4.2 Example 2

An example about investment alternatives for a multi-attribute decision-making problem studied in [43, 47, 48, 49, 62] is used to demonstrate the applicability of the proposed approach under interval neutrosophic environment.

An investment company wants to invest an amount of money in the best option. There are four possible alternatives to invest the money:

(1) A_1 is a car company;
(2) A_2 is a food company;
(3) A_3 is a computer company;
(4) A_4 is an arms company.

The proposed decision making method can be arranged in the following steps.

Step 1: Determination the most important criteria

The company must take a decision according to the three attributes as follows:

(1) G_1 is the risk;
(2) G_2 is the growth;
(3) G_3 is the environmental impact.

Step 2: Data pre-processing and Construction of the decision matrix with interval neutrosophic form

We obtain the following interval neutrosophic decision matrix based on the experts' assessment:

Table 6. Decision matrix with interval neutrosophic number

$d_S=\left\langle\left[T_{ij}^{L},T_{ij}^{U}\right],\left[I_{ij}^{L},I_{ij}^{U}\right],\left[F_{ij}^{L},F_{ij}^{U}\right]\right\rangle_{4\times3}$ =

	C_1	C_2	C_3
A_1	⟨[0.4,0.5], [0.2,0.3], [0.3,0.4]⟩	⟨[0.4,0.6], [0.1,0.3], [0.2,0.4]⟩	⟨[0.4,0.5], [0.7,0.8], [0.7,0.9]⟩
A_2	⟨[0.6,0.7], [0.1,0.2], [0.2,0.3]⟩	⟨[0.6,0.7], [0.1,0.2], [0.2,0.3]⟩	⟨[0.8,0.9], [0.5,0.7], [0.3,0.6]⟩
A_3	⟨[0.3,0.6], [0.2,0.3], [0.3,0.4]⟩	⟨[0.5,0.6], [0.2,0.3], [0.3,0.4]⟩	⟨[0.7,0.9], [0.6,0.8], [0.4,0.5]⟩
A_4	⟨[0.7,0.8], [0.0,0.1], [0.1,0.2]⟩	⟨[.6,.7], [0.1,0.2], [0.1,0.3]⟩	⟨[0.8,0.9], [0.6,0.7], [0.6,0.7]⟩

(28)

Step 3: Determination of the AAO

Using AAO, the decision matrix (see the table 7) in SVNS form is presented as follows:

Table7: single valued neutrosophic decision matrix

	C_1	C_2	C_3
A_1	⟨0.45,0.25,0.35⟩	⟨0.50,0.20,0.30⟩	⟨0.45,0.75,0.80⟩
A_2	⟨0.65,0.15,0.25⟩	⟨0.65,0.15,0.25⟩	⟨0.85,0.60,0.45⟩
A_3	⟨0.45,0.25,0.35⟩	⟨0.55,0.25,0.35⟩	⟨0.80,0.70,0.45⟩
A_4	⟨0.75,0.05,0.15⟩	⟨0.65,0.15,0.20⟩	⟨0.85,0.65,0.65⟩

(29)

Step 4: Determination of the weights of attribute

Entropy value E_j of the j-th (j = 1, 2, 3) attributes can be determined from the decision matrix d_S (12) and the equation (14). The obtained values are presented as follows: E_1 = 0.4400, E_2 = 0.4613, E_3 = 0.5413.

Then the entropy weights w_1, w_2, w_3 of the attributes are obtained from the eqation (15) and the obtained values are presented as follows: w_1 = 0.3596, w_2 = 0.3459, w_3 = 0.2945 such that $\sum_{j=1}^{4} w_j = 1$

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS)

The ideal interval neutrosophic estimates reliability solution (IINERS) is presented as follows.

$$Q_S^+ = \left\langle q_{S_1}^+,\ q_{S_2}^+, q_{S_3}^+ \right\rangle =$$

$$\left[\begin{array}{l} \left\langle \max\limits_i \{T_{i1}\}, \min\limits_i \{I_{i1}\}, \min\limits_i \{F_{i1}\} \right\rangle, \left\langle \max\limits_i \{T_{i2}\}, \min\limits_i \{I_{i2}\}, \min\limits_i \{F_{i2}\} \right\rangle, \\ \left\langle \max\limits_i \{T_{i3}\}, \min\limits_i \{I_{i3}\}, \min\limits_i \{F_{i3}\} \right\rangle \end{array}\right]$$

$$= \left[\langle 0.75, 0.05, 0.15\rangle, \langle 0.65, 0.15, 0.20\rangle, \langle 0.85, 0.60, 0.45\rangle\right]$$

The ideal interval neutrosophic estimates un-reliability solution (IINEURS) is presented as follows.

$$Q_S^- = \left\langle q_{S_1}^-,\ q_{S_2}^-, q_{S_3}^- \right\rangle =$$

$$\left[\begin{array}{l} \left\langle \min\limits_i \{T_{i1}\}, \max\limits_i \{I_{i1}\}, \max\limits_i \{F_{i1}\} \right\rangle, \left\langle \min\limits_i \{T_{i2}\}, \max\limits_i \{I_{i2}\}, \max\limits_i \{F_{i2}\} \right\rangle, \\ \left\langle \min\limits_i \{T_{i3}\}, \max\limits_i \{I_{i3}\}, \max\limits_i \{F_{i3}\} \right\rangle \end{array}\right]$$

$$= \left[\langle 0.45, 0.25, 0.35\rangle, \langle 0.50, 0.25, 0.35\rangle, \langle 0.45, 0.75, 0.80\rangle\right]$$

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Using equation (16), the interval neutrosophic grey relational coefficient of each alternative from IINERS can be obtained as the following matrix.

$$[G_{ij}^+]_{4\times 3} = \begin{bmatrix} 0.3913 & 0.6000 & 0.3333 \\ 0.6000 & 0.4737 & 1.0000 \\ 0.3913 & 0.5625 & 0.7500 \\ 1.0000 & 1.0000 & 0.6429 \end{bmatrix} \quad (30)$$

Similarly, from equation (17) the interval neutrosophic grey relational coefficient of each alternative from IINEURS is presented as the following matrix.

$$[G_{ij}^-]_{4\times 3} = \begin{bmatrix} 1.0000 & 0.8182 & 0.1.0000 \\ 0.5294 & 0.5625 & 0.3333 \\ 1.0000 & 0.9000 & 0.3750 \\ 0.3913 & 0.5294 & 0.4091 \end{bmatrix} \quad (31)$$

Step 7: Determine the degree of interval neutrosophic grey relational co-efficient of each alternative from IINERS and IINEURS. The required interval neutrosophic grey relational co-efficient corresponding to IINERS is obtained using equation (18) as follows:

$$G_1^+ = 0.4464, G_2^+ = 0.6741, G_3^+ = 0.5562, G_4^+ = 0.8548 \quad (32)$$

and corresponding to IINEURS is obtained with the help of equation (19) as follows:

$$G_1^- = 0.9371, G_2^- = 0.4831,\ G_3^- = 0.7813,\ G_4^- = 0.4443 \quad (33)$$

Step 8: The interval neutrosophic relative degree of each alternative from IINERS can be obtained with the help of equation (20) as follows:

$$R_1 = 0.3227, R_2 = 0.5825, R_3 = 0.4159, R_4 = 0.6580 \quad (34)$$

Step 9: The ranking order of all alternatives can be determined according to the decreasing order of the value of interval neutrosophic relative relational degree i.e. $R_4 > R_2 > R_3 > R_1$. It is seen that the highest value of interval neutrosophic relational degree is R_4. Therefore investment company must invest money in the best option A_4 (Arms company).

4.3 Comparision between the existing methods

The problem was studied by several methods [43, 47, 48, 49, 62]. Ye [47] proposed the similarity measures between INSs based on the relationship between similarity measures and distances and used the similarity measures between each alternative and the ideal alternative to establish a multicriteria decision making method for INSs. have two sets of rankings, $R_4 > R_2 > R_3 > R_1$ and $R_2 > R_4 > R_3 > R_1$ based two different similarity measures. Obviously, the two rankings in [47] conflict with each other. Ye [48] furthet proposed improved correlation coefficient for interval neutrosophic sets and obtained the ranking $R_2 > R_4 > R_3 > R_1$. In contrast, Zhang et al. [43] presented the aggregation operators for interval neutrosophic numbers and obtained the two different rankings $R_4 > R_1 > R_2 > R_3$ and $R_1 > R_4 > R_2 > R_3$. . Şahin, and Karabacak [62] suggested a set of axioms for the inclusion measure in a family of interval neutrosophic sets and proposed a simple and natural inclusion measure based on the normalized Hamming distance between interval neutrosophic sets. Şahin, and Karabacak [62] obtained the ranking $R_2 > R_4 > R_1 > R_3$. Chi and Liu [49] obtained the ranking $R_4 > R_2 > R_3 > R_1$. The above results reflect that the different methods yield different solution or rankings. This ensures that the study of interval neutrosophic decision making is interesting and challenging task. We can observe that our ranking order of the four alternatives and best choice are also in agreement with the results of Chi and Liu's externded Topsis method [49]. In addition, it is simpler in calculation process than of Chi and Liu's method [49].

5. Conclusion

INSs can be applied in dealing with problems having uncertain, imprecise, incomplete, and inconsistent information existing in real scientific and engineering applications. In this paper, we have introduced interval neutrosophic multi-attribute decision-making problem with completely unknown attribute weight information based on modified GRA. Here all the attribute weights information is unknown. Entropy based modified GRA analysis method has been introduced to solve this MADM problem. Interval neutrosophic grey relation coefficient has been proposed for solving multiple attribute decision-making problems. Finally, the effectiveness of the proposed approach is illustrated by solving two numerical examples. However, the authors hope that the concept presented here

will open new avenue of research in current neutrosophic decision-making arena. The main applications of this paper will be in the field of practical decision-making, medical diagnosis, pattern recognition, data mining, clustering analysis, etc.

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More on Neutrosophic Norms and Conorms

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Abstract**.** In 1995, Smarandache talked for the first time about neutrosophy and he defined one of the most important new mathematical tool which is a neutrosophic set theory as a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. He also defined the neutrosophic norm and conorms namely N-norm and N-conorm respectively. In this paper we give generating theorems for N-norm and N-conorm. Given an N-norm we can generate a class of N-norms and N-cnorms, and given an N-conorm we can generate a class of N-conorms and N-norms. We also give bijective generating theorems for N-norms and N-conorms.

Keywords: N-norm; N-conorm; generating theorem; bijective generating theorem.

1 Introduction

Since Zadeh [10] defined fuzzy set with *min* and *max* as the respective intersection and union operators, various alternatives operators have been proposed Dubois & Prade [2]; Yager [9]. The proposed operators are examples of the triangular norm and conorm (*t*-norm and *t*-conorm or *s*-norm) and hence fuzzy sets with these t-norms and s-norms as generalization of intersection and union are discussed in Klement [5], Waber [7], Wang [8] and Lowen [6]. In 2007 Alkhazaleh and Salleh [1] gave two generating theorems for *s*-norms and *t*-norms, namely given an *s*-norm we can generate a class of *s*-norms and *t*-norms, and given a *t*-norm we can generate a class of *t*-norms and *s*-norms. We also give two bijective generating theorems for *s*-norms and *t*-norms, that is given a bijective function under certain condition, we can generate new *s*-norm and *t*-norm from a given *s*-norm and also from a given *t*-norm. In 1995, Smarandache talked for the first time about neutrosophy and he in 1999 and 2005 [4, 3] defined one of the most important new mathematical tools which is a neutrosophic set theory as a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. He also presented the N-norms/N-conorms in neutrosophic logic and set as extensions of T-norms/T-conorms in fuzzy logic and set. In this paper we give two generating theorems for *N*-norms and *N*-conorms, namely given an *N*-norm we can generate a class of *N*-norms and *N*-conorms, and given a *N*-conorm we can generate a class of *N*-conorms and *N*-norms. We also give two bijective generating theorems for *N*-norms and *N*-conorms, that is given a bijective function under certain condition, we can generate new N-norm and N-conorm from a given N-norm and also from a given N-conorm.

2 Preliminaries

In this section, we recall some basic notions in fuzzy set and neutrosophic set theory.
For a fuzzy set we have the following s-norm and t-norm:

Definition 2.1 *The function* $s:[0,1]\times[0,1]\to[0,1]$ *is called an s-norm if it satisfies the following four requirements:*
Axiom s1. $s(x,y)=s(y,x)$ *(commutative condition).*
Axiom s2. s(s(x, y), z) = s(x, s(y, z)) (associative condition).
Axiom s3. If $x_1\geq x_2$ *and* $y_1\geq y_2$, *then* $s(x_1,y_1)\geq s(x_2,y_2)$ *(nondecreasing condition).*
Axiom s4. s(1, 1) = 1, s(x, 0) = s(0, x) = x (boundary condition).

Definition 2.2 *The function* $t:[0,1]\times[0,1]\to[0,1]$ *is called a* ***t-norm*** *if it satisfies the following four requirements:*
Axiom t1. t(x, y) = t(y, x) (commutative condition).
Axiom t2. t(t(x, y), z) = t(x, t(y, z)) (associative condition).
Axiom t3. If $x_1\geq x_2$ *and* $y_1\geq y_2$, *then* $t(x_1,y_1)\geq t(x_2,y_2)$ *(nondecreasing condition).*
Axiom t4 t(x, 1) = x (boundary condition).

	***s*-norm**	***t*-norm**
Basic	max(x,y)	min(x,y)
Bounded	$\min(x+y,1)$	$\max(x+y-1,0)$
Algebraic	$x+y-xy$	xy

Definition 2.3 [6] *A neutrosophic set* A *on the universe of discourse* X *is defined as*

$A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$ *where* T, I, $F : X \to]^{-}0,1^{+}[$ *and* $^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}$.

As a generalization of T-norm and T-conorm from the Fuzzy Logic and Set, Smarandache introduced the **N-norms and N-conorms** for the Neutrosophic Logic and Set. He defined a *partial relation order* on the neutrosophic set/logic in the following way:

$x(T_1, I_1, F_1) \le y(T_2, I_2, F_2)$ iff $T_1 \le T_2$, $I_1 \ge I_2$, $F_1 \ge F_2$ for crisp components. And, in general, for subunitary set components:

$$x(T_1, I_1, F_1) \le y(T_2, I_2, F_2) \text{ iff}$$
$$\inf T_1 \le \inf T_2, \ \sup T_1 \le \sup T_2,$$
$$\inf I_1 \ge \inf I_2, \ \sup I_1 \ge \sup I_2,$$
$$\inf F_1 \ge \inf F_2, \ \sup F_1 \ge \sup F_2.$$

Definition 2.4 [4] N-norms

$N_n : (\ ]^{-}0,1^{+}[\times]^{-}0,1^{+}[\times]^{-}0,1^{+}[\ )^2 \to (\ ]^{-}0,1^{+}[\times]^{-}0,1^{+}[\times]^{-}0,1^{+}[\ )$

$N_n(x(T_1,I_1,F_1), y(T_2,I_2,F_2)) = (N_nT(x,y), N_nI(x,y), N_nF(x,y))$,

where $N_nT(.,.)$, $N_nI(.,.)$, $N_nF(.,.)$ *are the truth /membership, indeterminacy, and respectively falsehood /nonmembership components.* N_n *have to satisfy, for any* x, y, z *in the neutrosophic logic/set* M *of the universe of discourse* U, *the following axioms:*

a) Boundary Conditions: $N_n(x,0) = 0$, $N_n(x,1) = x$.

b) Commutativity: $N_n(x, y) = N_n(y, x)$.

c) Monotonicity: If $x \le y$, *then* $N_n(x, z) \le N_n(y, z)$.

d) Associativity: $N_n(N_n(x, y), z) = N_n(x, N_n(y, z))$.

N_n *represent the and operator in neutrosophic logic, and respectively the intersection operator in neutrosophic set theory.*

Definition 2.5 [4] N-conorms

$N_c : (\ ]^{-}0,1^{+}[\times]^{-}0,1^{+}[\times]^{-}0,1^{+}[\ )^2 \to (\ ]^{-}0,1^{+}[\times]^{-}0,1^{+}[\times]^{-}0,1^{+}[\ )$

$N_c(x(T_1,I_1,F_1), y(T_2,I_2,F_2)) = (N_cT(x,y), N_cI(x,y), N_cF(x,y))$,

where $N_cT(.,.)$, $N_cI(.,.)$, $N_cF(.,.)$ are the truth /membership, indeterminacy, and respectively falsehood /nonmembership components. N_c have to satisfy, for any x, y, z in the neutrosophic logic/set M of the universe of discourse U, the following axioms:

a) Boundary Conditions: $N_c(x,0) = x$, $N_c(x,1) = 1$.

b) Commutativity: $N_c(x, y) = N_c(y, x)$.

c) Monotonicity: If $x \le y$, *then* $N_c(x, z) \le N_c(y, z)$.

d) Associativity: $N_c(N_c(x, y), z) = N_c(x, N_c(y, z))$.

A general example of N-norm would be this.

Let $x(T_1, I_1, F_1)$ and $y(T_1, I_1, F_1)$ be in the neutrosophic set/logic M. Then: $N_n(x, y) = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2)$.

A general example of N-conorm would be this. Let $x(T_1, I_1, F_1)$ and $y(T_1, I_1, F_1)$ be in the neutrosophic set/logic M. Then: $N_c(x, y) = (T_1 \vee T_2, I_1 \wedge I_2, F_1 \wedge F_2)$ where the "/\" operator, acting on two (standard or non-standard) subunitary sets, is a N-norm (verifying the above N-norms axioms); while the "\/" operator, also acting on two (standard or non-standard) subunitary sets, is a N-conorm (verifying the above N-conorms axioms). For example, /\ can be the Algebraic Product T-norm/N-norm, so T1/\T2 = T1·T2; and \/ can be the Algebraic Product T-conorm/N-conorm, so T1\/T2 = T1+T2-T1·T2. Or /\ can be any T-norm/N-norm, and \/ any T-conorm/N-conorm from the above and below; for example the easiest way would be to consider the min for crisp components (or inf for subset components) and respectively max for crisp components (or sup for subset components).

In the folowing we recall some theorems and corollaries given by Shawkat and Salleh 2007 in [1]

Theorem 2.6 *For any s-norm* $s(x, y)$ *and for all* $\alpha \ge 1$, *we get the following s-norms and t-norms*:

1. $S_\alpha^s(x,y) = \sqrt[\alpha]{s(x^\alpha, y^\alpha)}$,
2. $T_\alpha^s(x,y) = 1 - \sqrt[\alpha]{s((1-x)^\alpha, (1-y)^\alpha)}$.

Theorem 2.7 *For any t-norm* $t(x, y)$ *and for all* $\alpha > 1$, *we get the following t-norms and s-norms*:

1. $T_\alpha^t(x,y) = \sqrt[\alpha]{t(x^\alpha, y^\alpha)}$,
2. $S_\alpha^t(x,y) = 1 - \sqrt[\alpha]{t((1-x)^\alpha, (1-y)^\alpha)}$

Theorem 2.8 *Let* $f, g : [0,1] \to [0,1]$ *be bijective functions such that* $f(0) = 0$, $f(1) = 1$, $g(0) = 1$ *and* $g(1) = 0$. *For any s-norm* $s(x, y)$ *we get the following s-norm and t-norm*:

1. $S_f^s(x,y) = f^{-1}\left[s\left(f(x), f(y)\right)\right]$,
2. $T_g^s(x,y) = g^{-1}\left[s\left(g(x), g(y)\right)\right]$.

Corollary 2.9 *Let* $f(x) = \sin\frac{\pi}{2}x$ *and* $g(x) = \cos\frac{\pi}{2}x$ *then*

1. $S_{\sin}^s(x,y) = \frac{2}{\pi}\sin^{-1} s\left(\sin\frac{\pi}{2}x, \sin\frac{\pi}{2}y\right)$ is an *s*-norm
2. $T_{\cos}^s(x,y) = \frac{2}{\pi}\cos^{-1} s\left(\cos\frac{\pi}{2}x, \cos\frac{\pi}{2}y\right)$ is a *t*-norm

Theorem 2.10 *Let* $f, g : [0,1] \to [0,1]$ *be bijective functions such that* $f(0) = 0$, $f(1) = 1$, $g(0) = 1$ *and* $g(1) = 0$.

For any t-norm $t(x,y)$ *we get the following t-norm and s-norm*:

1. $T_f^t(x,y) = f^{-1}\left[t\left(f(x),f(y)\right)\right]$,
2. $S_g^t(x,y) = g^{-1}\left[t\left(g(x),g(y)\right)\right]$

Corollary 2.10 *Let f(x)=* $\sin\frac{\pi}{2}x$ *and g(x)=* $\cos\frac{\pi}{2}x$ *then*

1. $T_{\sin}^t(x,y) = \frac{2}{\pi}\sin^{-1}\left(t\left(\sin\frac{\pi}{2}x,\sin\frac{\pi}{2}y\right)\right)$ is a *t*-norm
2. $S_{\cos}^t(x,y) = \frac{2}{\pi}\cos^{-1}\left(t\left(\cos\frac{\pi}{2}x,\cos\frac{\pi}{2}y\right)\right)$ is an *s*-norm

3 Generating Theorems

In this section we give two generating theorems to generate N-norms and N-conorms by using any N-norms and N-conorms. Without loss of generality, we will rewrite the Smarandache's N-norm and N-conorm as it follows:

Definition 3.1

$T_n: (\,]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[\,)^2 \to (\,]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[\,)$

$T_n(x(T,I,F),\, y(T,I,F)) = \left(t(x_T,y_T),\, s(x_I,y_I),\, s(x_F,y_F)\right)$,

where $t(x_T,y_T)$, $s(x_I,y_I)$, $s(x_F,y_F)$ are the truth /membership, indeterminacy, and respectively falsehood /nonmembership components and *s* and *t* are the fuzzy s-norm and fuzzy t-norm respectively. T_n have to satisfy, for any x, y, z in the neutrosophic logic/set M of the universe of discourse U, the following axioms:

a) Boundary Conditions: $T_n(x,0) = 0,\ T_n(x,1) = x.$

b) Commutativity: $T_n(x,\, y) = T_n(y,\, x).$

c) Monotonicity: If $x \le y$, *then* $T_n(x,\, z) \le T_n(y,\, z).$

d) Associativity: $T_n\left(T_n(x,\, y),\, z\right) = T_n\left(x,\, T_n(y,\, z)\right).$

Definition 3.2 N-conorms

$S_n: (\,]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[\,)^2 \to (\,]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[\,)$

$S_n(x(T,I,F),\, y(T,I,F)) = \left(s(x_T,y_T),\, t(x_I,y_I),\, t(x_F,y_F)\right)$,

where $s(x_T,y_T)$, $t(x_I,y_I)$, $t(x_F,y_F)$ are the truth /membership, indeterminacy, and respectively falsehood /nonmembership components and *s* and *t* are the fuzzy s-norm and fuzzy t-norm respectively. S_n have to satisfy, for any x, y, z in the neutrosophic logic/set M of the universe of discourse U, the following axioms:

a) Boundary Conditions: $S_n(x,0) = x,\ S_n(x,1) = 1.$

b) Commutativity: $S_n(x,\, y) = S_n(y,\, x).$

c) Monotonicity: If $x \le y$, *then* $S_n(x,\, z) \le S_n(y,\, z).$

d) Associativity: $S_n\left(S_n(x,\, y),\, z\right) = S_n\left(x,\, S_n(y,\, z)\right).$

From now we use the following notation for N-norm and N-conorm respectively $T_n-(x,y)$ and $S_n-(x,y)$.

Remark: We will use the following border: $0(0,1,1)$ and $1(1,0,0)$.

Theorem 3.3. *For any* $S_n-(x,y)$ *and for all* $\alpha \ge 1$, by *using any fuzzy union s-norm we get the following* $S_n-(x,y)$ *and* $T_n-(x,y)$:

$$1.\ S_{n\,s}^{\alpha}(x,y) = \left\langle \begin{array}{l} \sqrt[\alpha]{s(x_T{}^{\alpha},y_T{}^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-x_I)^{\alpha},(1-y_I)^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-x_F)^{\alpha},(1-y_F)^{\alpha})} \end{array} \right\rangle$$

$$2.\ T_{n\,s}^{\alpha}(x,y) = \left\langle \begin{array}{l} 1-\sqrt[\alpha]{s\left(\left(1-x_T{}^{\alpha}\right),\left(1-y_T{}^{\alpha}\right)\right)}, \\ \sqrt[\alpha]{s\left(x_I{}^{\alpha},y_I{}^{\alpha}\right)}, \\ \sqrt[\alpha]{s\left(x_F{}^{\alpha},y_F{}^{\alpha}\right)} \end{array} \right\rangle.$$

Where *s* any s-norm (fuzzy union).

Proof. 1.

Axiom 1.

$$S_{n\,s}^{\alpha}(0,x) = \left\langle \begin{array}{l} \sqrt[\alpha]{s(0^{\alpha},x_T{}^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-1)^{\alpha},(1-x_I)^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-1)^{\alpha},(1-x_F)^{\alpha})} \end{array} \right\rangle = x\left(x_T,x_I,x_F\right).$$

$$S_{n\,s}^{\alpha}(1,x) = \left\langle \begin{array}{l} \sqrt[\alpha]{s(1^{\alpha},x_T{}^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-0)^{\alpha},(1-x_I)^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-0)^{\alpha},(1-x_F)^{\alpha})} \end{array} \right\rangle = 1(1,0,0)$$

Axiom 2.

$$S_{n\,s}^{\alpha}(x,y) = \left\langle \begin{array}{l} \sqrt[\alpha]{s(x_T{}^{\alpha},y_T{}^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-x_I)^{\alpha},(1-y_I)^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-x_F)^{\alpha},(1-y_F)^{\alpha})} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \sqrt[\alpha]{s(y_T{}^{\alpha},x_T{}^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-y_I)^{\alpha},(1-x_I)^{\alpha})}, \\ 1-\sqrt[\alpha]{s((1-y_F)^{\alpha},(1-x_F)^{\alpha})} \end{array} \right\rangle.$$

$$= S_{n\,s}^{\alpha}(y,x)$$

Axiom 3. Let $x(x_1, x_2, x_3) \le y(y_1, y_2, y_3)$ then $x_1 \le y_1, x_2 \ge y_2, x_3 \ge y_3$ and $s(x_1^\alpha, z_1^\alpha) \ge s(y_1^\alpha, z_1^\alpha)$ which implies $\sqrt[\alpha]{s(x_1^\alpha, z_1^\alpha)} \ge \sqrt[\alpha]{s(y_2^\alpha, z_2^\alpha)}$. (1)

Also we have $(1-x_2)^\alpha \le (1-y_2)^\alpha$ then $s((1-x_2)^\alpha, (1-z_2)^\alpha) \le s((1-y_2)^\alpha, (1-z_2)^\alpha)$, which implies that

$$1-\sqrt[\alpha]{s((1-x_2)^\alpha, (1-z_2)^\alpha)} \ge 1-\sqrt[\alpha]{s((1-y_2)^\alpha, (1-z_2)^\alpha)} \quad (2)$$

And we have $(1-x_3)^\alpha \le (1-y_3)^\alpha$ then $s((1-x_3)^\alpha, (1-z_3)^\alpha) \le s((1-y_3)^\alpha, (1-z_3)^\alpha)$, which implies that

$$1-\sqrt[\alpha]{s((1-x_3)^\alpha, (1-z_3)^\alpha)} \ge 1-\sqrt[\alpha]{s((1-y_3)^\alpha, (1-z_3)^\alpha)} \quad (3)$$

From (1), (2) and (3) we have $S_n^\alpha(x,z) \ge S_n^\alpha(y,z)$.

Axiom 4.

$$S_n^\alpha\left(\underset{s}{S_n^\alpha}(x,y), z\right) = S_n^\alpha\left(\left\langle \begin{array}{l} \sqrt[\alpha]{s(x_T{}^\alpha, y_T{}^\alpha)}, \\ 1-\sqrt[\alpha]{s((1-x_I)^\alpha, (1-y_I)^\alpha)}, \\ 1-\sqrt[\alpha]{s((1-x_F)^\alpha, (1-y_F)^\alpha)} \end{array} \right\rangle, z\right)$$

$$= \left\langle \begin{array}{l} \sqrt[\alpha]{s\left(\sqrt[\alpha]{s(x_T{}^\alpha, y_T{}^\alpha)}^\alpha, z_T{}^\alpha\right)}, \\ 1-\sqrt[\alpha]{s\left(\left\lfloor 1-\left(1-\sqrt[\alpha]{s((1-x_I)^\alpha, (1-y_I)^\alpha)}\right)\right\rfloor^\alpha, (1-z_I)^\alpha\right)}, \\ 1-\sqrt[\alpha]{s\left(\left\lfloor 1-\left(1-\sqrt[\alpha]{s((1-x_F)^\alpha, (1-y_F)^\alpha)}\right)\right\rfloor^\alpha, (1-z_F)^\alpha\right)} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \sqrt[\alpha]{s\left(s(x_T{}^\alpha, y_T{}^\alpha), z_T{}^\alpha\right)}, \\ 1-\sqrt[\alpha]{s\left(s((1-x_I{}^\alpha), (1-y_I{}^\alpha)), (1-z_I{}^\alpha)\right)}, \\ 1-\sqrt[\alpha]{s\left(s((1-x_F{}^\alpha), (1-y_F{}^\alpha)), (1-z_F{}^\alpha)\right)} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \sqrt[\alpha]{s\left(x_T{}^\alpha, s(y_T{}^\alpha, z_T{}^\alpha)\right)}, \\ 1-\sqrt[\alpha]{s\left((1-x_I{}^\alpha), s((1-y_I{}^\alpha), (1-z_I{}^\alpha))\right)}, \\ 1-\sqrt[\alpha]{s\left((1-x_F{}^\alpha), s((1-y_F{}^\alpha), (1-z_F{}^\alpha))\right)} \end{array} \right\rangle$$

$$= S_n^\alpha\left(x, S_n^\alpha(y,z)\right)$$

Therefore $\underset{s}{S_n^\alpha}(x,y)$ is an N-conorm.

2. The proof is similar to Proof 1. □

Theorem 3.4. *For any $T_n-(x,y)$ and for all $\alpha \ge 1$, by using any fuzzy intersection t-norm we get the following $S_n-(x,y)$ and $T_n-(x,y)$:*

$$1.\ \underset{t}{S_n^\alpha}(x,y) = \left\langle \begin{array}{l} 1-\sqrt[\alpha]{t\left((1-x_T{}^\alpha), (1-y_T{}^\alpha)\right)}, \\ \sqrt[\alpha]{t\left(x_I{}^\alpha, y_I{}^\alpha\right)}, \\ \sqrt[\alpha]{t\left(x_F{}^\alpha, y_F{}^\alpha\right)} \end{array} \right\rangle$$

$$2.\ \underset{t}{T_n^\alpha}(x,y) = \left\langle \begin{array}{l} \sqrt[\alpha]{t\left(x^{\ \alpha}\ y_T{}^\alpha\right)}, \\ 1-\sqrt[\alpha]{t\left((1-x_I{}^\alpha), (1-y_I{}^\alpha)\right)}, \\ 1-\sqrt[\alpha]{t\left((1-x_F{}^\alpha), (1-y_F{}^\alpha)\right)} \end{array} \right\rangle.$$

Where *t* any t-norm (fuzzy intersection).

Proof. The proof is similar to Proof of theorem 3.3. □

4. Bijective Generating Theorems

In this section we give two generating theorems to generate *N-norms* and *N-conorms* from any *N-norms* and *N-conorms*. By these theorems we can generate infinitely many *N-norms* and *N-conorms* by using two bijective functions with certain conditions.

Theorem 4.1. *Let $f, g:[0,1]\to[0,1]$ be bijective functions such that $f(0)=0$, $f(1)=1$, $g(0)=1$ and $g(1)=0$. For any $S_n-(x,y)$ and by using any fuzzy union s-norm we get the following $S_n-(x,y)$ and $T_n-(x,y)$:*

$$1.\ \underset{f,g}{S_n^s}(x,y) = \left\langle \begin{array}{l} f^{-1}\left\lfloor s\left(f(x_T), f(y_T)\right)\right\rfloor, \\ g^{-1}\left[s\left(g(x_I), g(y_I)\right)\right], \\ g^{-1}\left[s\left(g(x_F), g(y_F)\right)\right] \end{array} \right\rangle,$$

$$2.\ \underset{f,g}{T_n^s}(x,y) = \left\langle \begin{array}{l} g^{-1}\left[s\left(g(x_T), g(y_T)\right)\right], \\ f^{-1}\left[s\left(f(x_I), f(y_I)\right)\right], \\ f^{-1}\left[s\left(f(x_F), f(y_F)\right)\right] \end{array} \right\rangle.$$

Proof. 1.

Axiom 1.

$$S_n^s(x,0)=\left\langle \begin{array}{l} f^{-1}\left[s\left(f(x_T),f(0)\right)\right],\\ g^{-1}\left[s\left(g(x_I),g(1)\right)\right],\\ g^{-1}\left[s\left(g(x_F),g(1)\right)\right] \end{array} \right\rangle = x.$$

$$S_n^s(x,1)=\left\langle \begin{array}{l} f^{-1}\left[s\left(f(x_T),f(1)\right)\right],\\ g^{-1}\left[s\left(g(x_I),g(0)\right)\right],\\ g^{-1}\left[s\left(g(x_F),g(0)\right)\right] \end{array} \right\rangle = 1(1,0,0).$$

Axiom 2.

$$S_n^s(x,y)=\left\langle \begin{array}{l} f^{-1}\left[s\left(f(x_T),f(y_T)\right)\right],\\ g^{-1}\left[s\left(g(x_I),g(y_I)\right)\right],\\ g^{-1}\left[s\left(g(x_F),g(y_F)\right)\right] \end{array} \right\rangle$$

$$=\left\langle \begin{array}{l} f^{-1}\left[s\left(f(y_T),f(x_T)\right)\right],\\ g^{-1}\left[s\left(g(y_I),g(x_I)\right)\right],\\ g^{-1}\left[s\left(g(y_F),g(x_F)\right)\right] \end{array} \right\rangle = S_n^s(y,x).$$

Axiom 3. Let $x \le y$. Since f is bijective on the interval $[0,1]$ and by Axiom s3 we have

$s\left(f(x_T),f(z_T)\right) \le s\left(f(y_T),f(z_T)\right)$ then

$f^{-1}\left[s\left(f(x_T),f(z_T)\right)\right] \le f^{-1}\left[s\left(f(y_T),f(z_T)\right)\right]$ (1)

Also since g is bijective on the interval $[0,1]$ and by Axiom t3 we have

$s\left(g(x_I),g(z_I)\right) \ge s\left(g(y_I),g(z_I)\right)$ then

$g^{-1}\left[s\left(g(x_I),g(z_I)\right)\right] \ge g^{-1}\left[s\left(g(y_I),g(z_I)\right)\right]$ (2)

And

$s\left(g(x_F),g(z_F)\right) \ge s\left(g(y_F),g(z_F)\right)$ then

$g^{-1}\left[s\left(g(x_F),g(z_F)\right)\right] \ge g^{-1}\left[s\left(g(y_F),g(z_F)\right)\right]$ (3)

From (1), (2) and (3) we have $S_n^s(x,z) \le S_n^s(y,z)$

Axiom 4.

$$S_n^s\left(S_n^s(x,y),z\right)=S_n^s\left(\left\langle \begin{array}{l} f^{-1}\left[s\left(f(x_T),f(y_T)\right)\right],\\ g^{-1}\left[s\left(g(x_I),g(y_I)\right)\right],\\ g^{-1}\left[s\left(g(x_F),g(y_F)\right)\right] \end{array} \right\rangle,z\right)$$

$$=\left\langle \begin{array}{l} f^{-1}\left(f\left(f^{-1}\left[s\left(f(x_T),f(y_T)\right)\right]\right),f(z_T)\right),\\ g^{-1}\left(g\left(g^{-1}\left[s\left(g(x_I),g(y_I)\right)\right]\right),g(z_I)\right),\\ g^{-1}\left(g\left(g^{-1}\left[s\left(g(x_F),g(y_F)\right)\right]\right),g(z_F)\right) \end{array} \right\rangle$$

$$=\left\langle \begin{array}{l} f^{-1}\left(s\left(f(x_T),f(y_T)\right),f(z_T)\right),\\ g^{-1}\left(s\left(g(x_I),g(y_I)\right),g(z_I)\right),\\ g^{-1}\left(s\left(g(x_F),g(y_F)\right),g(z_F)\right) \end{array} \right\rangle$$

$$=\left\langle \begin{array}{l} f^{-1}\left(f(x_T),s\left(f(y_T),f(z_T)\right)\right),\\ g^{-1}\left(g(x_I),s\left(g(y_I),g(z_I)\right)\right),\\ g^{-1}\left(g(x_F),s\left(g(y_F),g(z_F)\right)\right) \end{array} \right\rangle$$

$$= S_n^s\left(x,S_n^s(y,z)\right).$$

Therefore $S_n^s(x,y)$ is an $S_n-(x,y)$ □

2. The Proof is similar to Proof 1.

Corollary 4.2. *Let* $f(x)=\sin\frac{\pi}{2}x$ *and* $g(x)=\cos\frac{\pi}{2}x$ *then*

1. $$S_{n\,\sin,\cos}^s(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}s\left(\sin\frac{\pi}{2}x_T,\sin\frac{\pi}{2}y_T\right),\\ \frac{2}{\pi}\cos^{-1}s\left(\cos\frac{\pi}{2}x_I,\cos\frac{\pi}{2}y_I\right),\\ \frac{2}{\pi}\cos^{-1}s\left(\cos\frac{\pi}{2}x_F,\cos\frac{\pi}{2}y_F\right) \end{array} \right\rangle$$

is an $S_n-(x,y)$

$$2.\ T_{n}^{s}(x,y)=\underset{\sin,\cos}{\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}s\left(\cos\frac{\pi}{2}x_T,\cos\frac{\pi}{2}y_T\right),\\ \frac{2}{\pi}\sin^{-1}s\left(\sin\frac{\pi}{2}x_I,\sin\frac{\pi}{2}y_I\right),\\ \frac{2}{\pi}\sin^{-1}s\left(\sin\frac{\pi}{2}x_F,\sin\frac{\pi}{2}y_F\right) \end{array}\right\rangle}$$

is a $T_n-(x,y)$

Theorem 4.3. *Let* $f, g:[0,1]\to[0,1]$ *be bijective functions such that* $f(0)=0$, $f(1)=1$, $g(0)=1$ *and* $g(1)=0$. *For any* $T_n-(x,y)$ *and by using any fuzzy intersection t-norm we get the following* $S_n-(x,y)$ *and* $T_n-(x,y)$:

$$1.\ S_{n}^{t}(x,y)=\underset{f,g}{\left\langle \begin{array}{l} g^{-1}\left[t\left(g(x_T),g(y_T)\right)\right],\\ f^{-1}\left[t\left(f(x_I),f(y_I)\right)\right],\\ f^{-1}\left[t\left(f(x_F),f(y_F)\right)\right] \end{array}\right\rangle},$$

$$2.\ T_{n}^{t}(x,y)=\underset{f,g}{\left\langle \begin{array}{l} f^{-1}\left[t\left(f(x_T),f(y_T)\right)\right],\\ g^{-1}\left[t\left(g(x_I),g(y_I)\right)\right],\\ g^{-1}\left[t\left(g(x_F),g(y_F)\right)\right] \end{array}\right\rangle}.$$

Proof. The proof is similar to Proof of theorem 4.1. □

Corollary 4.4. *Let* $f(x)=\sin\frac{\pi}{2}x$ *and* $g(x)=\cos\frac{\pi}{2}x$ *then*

$$1.\ S_{\sin,\cos}^{t}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}t\left(\cos\frac{\pi}{2}x_T,\cos\frac{\pi}{2}y_T\right),\\ \frac{2}{\pi}\sin^{-1}t\left(\sin\frac{\pi}{2}x_I,\sin\frac{\pi}{2}y_I\right),\\ \frac{2}{\pi}\sin^{-1}t\left(\sin\frac{\pi}{2}x_F,\sin\frac{\pi}{2}y_F\right) \end{array}\right\rangle$$

is an $S_n-(x,y)$

$$2.\ T_{\sin,\cos}^{t}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}t\left(\sin\frac{\pi}{2}x_T,\sin\frac{\pi}{2}y_T\right),\\ \frac{2}{\pi}\cos^{-1}t\left(\cos\frac{\pi}{2}x_I,\cos\frac{\pi}{2}y_I\right),\\ \frac{2}{\pi}\cos^{-1}t\left(\cos\frac{\pi}{2}x_F,\cos\frac{\pi}{2}y_F\right) \end{array}\right\rangle$$

is a $T_n-(x,y)$

5. Examples

In this section we generate some new $S_n-(x,y)$ and $T_n-(x,y)$ from existing $S_n-(x,y)$ and $T_n-(x,y)$ using the Generating Theorems and the Bijective Generating Theorems.

5. 1. BOUNDED SUM GENERATING CLASSES

New $S_n-(x,y)$ and $T_n-(x,y)$ from the bounded sum *s*-norm.

$$S_{n}^{\alpha}(x,y)=\underset{s}{\left\langle \begin{array}{l} \sqrt[\alpha]{\min({x_T}^{\alpha}+{y_T}^{\alpha},1)},\\ 1-\sqrt[\alpha]{\min((1-x_I)^{\alpha}+(1-y_I)^{\alpha},1)},\\ 1-\sqrt[\alpha]{\min((1-x_F)^{\alpha}+(1-y_F)^{\alpha},1)} \end{array}\right\rangle}$$

$$T_{n}^{\alpha}(x,y)=\underset{bs}{\left\langle \begin{array}{l} 1-\sqrt[\alpha]{\min\left((1-x_T)^{\alpha}+(1-\ _T)^{\alpha},1\right)},\\ \sqrt[\alpha]{\min\left({x_I}^{\alpha}+{y_I}^{\alpha},1\right)},\\ \sqrt[\alpha]{\min\left({x_F}^{\alpha}+{y_F}^{\alpha},1\right)} \end{array}\right\rangle}.$$

$$S_{\sin,\cos}^{bs}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}\left(\min\left(\left(\sin\frac{\pi}{2}x_T\right)+\left(\sin\frac{\pi}{2}y_T\right),1\right)\right),\\ \frac{2}{\pi}\cos^{-1}\left(\min\left(\left(\cos\frac{\pi}{2}x_I\right)+\left(\cos\frac{\pi}{2}y_I\right),1\right)\right),\\ \frac{2}{\pi}\cos^{-1}\left(\min\left(\left(\cos\frac{\pi}{2}x_F\right)+\left(\cos\frac{\pi}{2}y_F\right),1\right)\right) \end{array}\right\rangle$$

$$T_{n}^{bs}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}\left(\min\left(\left(\cos\frac{\pi}{2}x_T\right)+\left(\cos\frac{\pi}{2}y_T\right),1\right)\right), \\ \frac{2}{\pi}\sin^{-1}\left(\min\left(\left(\sin\frac{\pi}{2}x_I\right)+\left(\sin\frac{\pi}{2}y_I\right),1\right)\right), \\ \frac{2}{\pi}\sin^{-1}\left(\min\left(\left(\sin\frac{\pi}{2}x_F\right)+\left(\sin\frac{\pi}{2}y_F\right),1\right)\right) \end{array}\right\rangle.$$

$$S_{n}^{\alpha}(x,y)=\left\langle \begin{array}{l} \sqrt[\alpha]{\frac{x_T{}^{\alpha}+y_T{}^{\alpha}}{1+x_T{}^{\alpha}y_T{}^{\alpha}}}, \\ 1-\sqrt[\alpha]{\frac{(1-x_I)^{\alpha}+(1-y_I)^{\alpha}}{1+(1-x_I)^{\alpha}(1-y_I)^{\alpha}}}, \\ 1-\sqrt[\alpha]{\frac{(1-x_F)^{\alpha}+(1-y_F)^{\alpha}}{1+(1-x_F)^{\alpha}(1-y_F)^{\alpha}}} \end{array}\right\rangle$$

5.2. ALGEBRAIC SUM GENERATING CLASSES

New $S_n-(x,y)$ and $T_n-(x,y)$ from the algebraic sum *s*-norm.

$$S_{as}^{\alpha}(x,y)=\left\langle \begin{array}{l} \sqrt[\alpha]{x_T{}^{\alpha}+y_T{}^{\alpha}-x_T{}^{\alpha}.y_T{}^{\alpha}}, \\ 1-\sqrt[\alpha]{(1-x_I)^{\alpha}+(1-y_I)^{\alpha}-(1-x_I)^{\alpha}.(1-y_I)^{\alpha}}, \\ 1-\sqrt[\alpha]{(1-x_F)^{\alpha}+(1-y_F)^{\alpha}-(1-x_F)^{\alpha}.(1-y_F)^{\alpha}} \end{array}\right\rangle$$

$$T_{as}^{\alpha}(x,y)=\left\langle \begin{array}{l} 1-\sqrt[\alpha]{(1-x_T)^{\alpha}+(1-y_T)^{\alpha}-(1-x_T)^{\alpha}(1-y_T)^{\alpha}}, \\ \sqrt[\alpha]{x_I{}^{\alpha}+y_I{}^{\alpha}-x_I{}^{\alpha}.y_I{}^{\alpha}}, \\ \sqrt[\alpha]{x_F{}^{\alpha}+y_F{}^{\alpha}-x_F{}^{\alpha}.y_F{}^{\alpha}} \end{array}\right\rangle.$$

$$S_{\sin,\cos}^{as}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}\left(\left(\sin\frac{\pi}{2}x_T\right)+\left(\sin\frac{\pi}{2}y_T\right)-\left(\sin\frac{\pi}{2}x_T\right)\left(\sin\frac{\pi}{2}y_T\right)\right), \\ \frac{2}{\pi}\cos^{-1}\left(\left(\cos\frac{\pi}{2}x_I\right)+\left(\cos\frac{\pi}{2}y_I\right)-\left(\cos\frac{\pi}{2}x_I\right)\left(\cos\frac{\pi}{2}y_I\right)\right), \\ \frac{2}{\pi}\cos^{-1}\left(\left(\cos\frac{\pi}{2}x_F\right)+\left(\cos\frac{\pi}{2}y_F\right)-\left(\cos\frac{\pi}{2}x_F\right)\left(\cos\frac{\pi}{2}y_F\right)\right) \end{array}\right\rangle$$

$$T_{\sin,\cos}^{as}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}\left(\left(\cos\frac{\pi}{2}x_T\right)+\left(\cos\frac{\pi}{2}y_T\right)-\left(\cos\frac{\pi}{2}x_T\right)\left(\cos\frac{\pi}{2}y_T\right)\right), \\ \frac{2}{\pi}\sin^{-1}\left(\left(\sin\frac{\pi}{2}x_I\right)+\left(\sin\frac{\pi}{2}y_I\right)-\left(\sin\frac{\pi}{2}x_I\right)\left(\sin\frac{\pi}{2}y_I\right)\right), \\ \frac{2}{\pi}\sin^{-1}\left(\left(\sin\frac{\pi}{2}x_F\right)+\left(\sin\frac{\pi}{2}y_F\right)-\left(\sin\frac{\pi}{2}x_F\right)\left(\sin\frac{\pi}{2}y_F\right)\right) \end{array}\right\rangle.$$

5.3. EINSTEIN SUM GENERATING CLASSES

New $S_n-(x,y)$ and $T_n-(x,y)$ from the Einstein sum *s*-norm.

$$T_{es}^{\alpha}(x,y)=\left\langle \begin{array}{l} 1-\sqrt[\alpha]{\frac{(1-x_T)^{\alpha}+(1-y_T)^{\alpha}}{1+(1-x_T)^{\alpha}(1-y_T)^{\alpha}}}, \\ \sqrt[\alpha]{\frac{x_I{}^{\alpha}+y_I{}^{\alpha}}{1+x_I{}^{\alpha}y_I{}^{\alpha}}}, \\ \sqrt[\alpha]{\frac{x_F{}^{\alpha}+y_F{}^{\alpha}}{1+x_F{}^{\alpha}y_F{}^{\alpha}}} \end{array}\right\rangle.$$

$$S_{\sin,\cos}^{bs}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}\left(\frac{\left(\sin\frac{\pi}{2}x_T\right)+\left(\sin\frac{\pi}{2}y_T\right)}{1+\left(\sin\frac{\pi}{2}x_T\right)\left(\sin\frac{\pi}{2}y_T\right)}\right), \\ \frac{2}{\pi}\cos^{-1}\left(\frac{\left(\cos\frac{\pi}{2}x_I\right)+\left(\cos\frac{\pi}{2}y_I\right)}{1+\left(\cos\frac{\pi}{2}x_I\right)\left(\cos\frac{\pi}{2}y_I\right)}\right), \\ \frac{2}{\pi}\cos^{-1}\left(\frac{\left(\cos\frac{\pi}{2}x_F\right)+\left(\cos\frac{\pi}{2}y_F\right)}{1+\left(\cos\frac{\pi}{2}x_F\right)\left(\cos\frac{\pi}{2}y_F\right)}\right) \end{array}\right\rangle$$

$$T_{\sin,\cos}^{bs}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}\left(\frac{\left(\cos\frac{\pi}{2}x_T\right)+\left(\cos\frac{\pi}{2}y_T\right)}{1+\left(\cos\frac{\pi}{2}x_T\right)\left(\cos\frac{\pi}{2}y_T\right)}\right), \\ \frac{2}{\pi}\sin^{-1}\left(\frac{\left(\sin\frac{\pi}{2}x_I\right)+\left(\sin\frac{\pi}{2}y_I\right)}{1+\left(\sin\frac{\pi}{2}x_I\right)\left(\sin\frac{\pi}{2}y_I\right)}\right), \\ \frac{2}{\pi}\sin^{-1}\left(\frac{\left(\sin\frac{\pi}{2}x_F\right)+\left(\sin\frac{\pi}{2}y_F\right)}{1+\left(\sin\frac{\pi}{2}x_F\right)\left(\sin\frac{\pi}{2}y_F\right)}\right) \end{array}\right\rangle.$$

5. 4. BOUNDED PRODUCT GENERATING CLASSES

New $S_n-(x,y)$ and $T_n-(x,y)$ from the bounded product *t*-norm.

$$T_{n}^{\alpha}(x,y)=\left\langle \begin{array}{l} \sqrt[\alpha]{\max\left(x_T{}^{\alpha} \quad {}_T{}^{\alpha}-1,0\right)}, \\ 1-\sqrt[\alpha]{\max\left(\left(1-x_I\right)^{\alpha}+\left(1-y_I{}^{\alpha}\right)-1,0\right)}, \\ 1-\sqrt[\alpha]{\max\left(\left(1-x_F\right)^{\alpha}+\left(1-y_F\right)^{\alpha}-1,0\right)} \end{array} \right\rangle.$$

$$S_{n}^{\alpha}(x,y)=\left\langle \begin{array}{l} 1-\sqrt[\alpha]{\max\left(\left(1-x_T\right)^{\alpha}+\left(1- \quad {}_T\right)^{\alpha}-1,0\right)}, \\ \sqrt[\alpha]{\max\left(x_I{}^{\alpha}+y_I{}^{\alpha}-1,0\right)}, \\ \sqrt[\alpha]{\max\left(x_F{}^{\alpha}+y_F{}^{\alpha}-1,0\right)} \end{array} \right\rangle$$

$$T_{\sin,\cos}^{bp}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}\left(\max\left(\left(\sin\frac{\pi}{2}x_T\right)+\left(\sin\frac{\pi}{2}y_T\right)-1,0\right)\right), \\ \frac{2}{\pi}\cos^{-1}\left(\max\left(\left(\cos\frac{\pi}{2}x_I\right)+\left(\cos\frac{\pi}{2}y_I\right)-1,0\right)\right), \\ \frac{2}{\pi}\cos^{-1}\left(\max\left(\left(\cos\frac{\pi}{2}x_F\right)+\left(\cos\frac{\pi}{2}y_F\right)-1,0\right)\right) \end{array} \right\rangle.$$

$$S_{\sin,\cos}^{t}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}\left(\max\left(\left(\cos\frac{\pi}{2}x_T\right)+\left(\cos\frac{\pi}{2}y_T\right)-1,0\right)\right), \\ \frac{2}{\pi}\sin^{-1}\left(\max\left(\left(\sin\frac{\pi}{2}x_I+\sin\frac{\pi}{2}y_I\right)-1,0\right)\right), \\ \frac{2}{\pi}\sin^{-1}\left(\max\left(\left(\sin\frac{\pi}{2}x_F+\sin\frac{\pi}{2}y_F\right)-1,0\right)\right) \end{array} \right\rangle.$$

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5.5. EINSTEIN PRODUCT GENERATING CLASSES

New $S_n-(x,y)$ and $T_n-(x,y)$ from the Einstein product *t*-norm.

$$T_{ep}^{\alpha}(x,y)=\left\langle \begin{array}{l} \sqrt[\alpha]{\frac{x_T{}^{\alpha}y_T{}^{\alpha}}{2-\left(x_T{}^{\alpha}+y_T{}^{\alpha}-x_T{}^{\alpha}y_T{}^{\alpha}\right)}}, \\ 1\quad\sqrt[\alpha]{\frac{\left(1-x\right)^{\alpha}\left(1-y_I\right)^{\alpha}}{2-\left(\left(1-x_I\right)^{\alpha}+\left(1-y_I\right)^{\alpha}-\left(1-x_I\right)^{\alpha}\left(1-y_I\right)^{\alpha}\right)}}, \\ 1-\sqrt[\alpha]{\frac{\left(1-x_F\right)^{\alpha}\left(1-y_F\right)^{\alpha}}{2-\left(\left(1-x_F\right)^{\alpha}+\left(1-y_F\right)^{\alpha}-\left(1-x_F\right)^{\alpha}\left(1-y_F\right)^{\alpha}\right)}} \end{array} \right\rangle.$$

$$S_{ep}^{\alpha}(x,y)=\left\langle \begin{array}{l} 1-\sqrt[\alpha]{\frac{\left(1-x_T\right)^{\alpha}\left(1-y_T\right)^{\alpha}}{2-\left(\left(1-x_T\right)^{\alpha}+\left(1-y_T\right)^{\alpha}-\left(1-x_T\right)^{\alpha}\left(1-y_T\right)^{\alpha}\right)}}, \\ \sqrt[\alpha]{\frac{x_I{}^{\alpha}y^{\alpha}}{2-\left(x_I{}^{\alpha}+y_I{}^{\alpha}-x_I{}^{\alpha}y_I{}^{\alpha}\right)}}, \\ \sqrt[\alpha]{\frac{x_F{}^{\alpha}y_F{}^{\alpha}}{2-\left(x_F{}^{\alpha}+y_F{}^{\alpha}-x_F{}^{\alpha}y_F{}^{\alpha}\right)}} \end{array} \right\rangle$$

$$T_{\sin,\cos}^{ep}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\sin^{-1}\left(\frac{\left(\sin\frac{\pi}{2}x_T\right)\left(\sin\frac{\pi}{2}y_T\right)}{2-\left(\left(\sin\frac{\pi}{2}x_T\right)+\left(\sin\frac{\pi}{2}y_T\right)-\left(\sin\frac{\pi}{2}x_T\right)\left(\sin\frac{\pi}{2}y_T\right)\right)}\right), \\ \frac{2}{\pi}\cos^{-1}\left(\frac{\left(\cos\frac{\pi}{2}x_I\right)\left(\cos\frac{\pi}{2}y_I\right)}{2-\left(\left(\cos\frac{\pi}{2}x_I\right)+\left(\cos\frac{\pi}{2}y_I\right)-\left(\cos\frac{\pi}{2}x_I\right)\left(\cos\frac{\pi}{2}y_I\right)\right)}\right), \\ \frac{2}{\pi}\cos^{-1}\left(\frac{\left(\cos\frac{\pi}{2}x_F\right)\left(\cos\frac{\pi}{2}y_F\right)}{2-\left(\left(\cos\frac{\pi}{2}x_F\right)+\left(\cos\frac{\pi}{2}y_F\right)-\left(\cos\frac{\pi}{2}x_F\right)\left(\cos\frac{\pi}{2}y_F\right)\right)}\right) \end{array} \right\rangle$$

$$s_{\sin,\cos}^{ep}(x,y)=\left\langle \begin{array}{l} \frac{2}{\pi}\cos^{-1}\left(\frac{\left(\cos\frac{\pi}{2}x_T\right)\left(\cos\frac{\pi}{2}y_T\right)}{2-\left(\left(\cos\frac{\pi}{2}x_T\right)+\left(\cos\frac{\pi}{2}y_T\right)-\left(\cos\frac{\pi}{2}x_T\right)\left(\cos\frac{\pi}{2}y_T\right)\right)}\right), \\ \frac{2}{\pi}\sin^{-1}\left(\frac{\left(\sin\frac{\pi}{2}x_I\right)\left(\sin\frac{\pi}{2}y_I\right)}{2-\left(\left(\sin\frac{\pi}{2}x_I\right)+\left(\sin\frac{\pi}{2}y_I\right)-\left(\sin\frac{\pi}{2}x_I\right)\left(\sin\frac{\pi}{2}y_I\right)\right)}\right), \\ \frac{2}{\pi}\sin^{-1}\left(\frac{\left(\sin\frac{\pi}{2}x_F\right)\left(\sin\frac{\pi}{2}y_F\right)}{2-\left(\left(\sin\frac{\pi}{2}x_F\right)+\left(\sin\frac{\pi}{2}y_F\right)-\left(\sin\frac{\pi}{2}x_F\right)\left(\sin\frac{\pi}{2}y_F\right)\right)}\right) \end{array} \right\rangle$$

Remark. Note that for the *s*-norms max and drastic sum and *t*-norms min, algebraic product and drastic product we get the same norms

6 Conclusion

In this paper, we gave generating theorems for N-norm and N-conorm. By given any N-norm we generated a class of N-norms and N-cnorms, and by given any N-conorm we generated a class of N-conorms and N-norms. We also gave bijective generating theorems for N-norms and N-conorms.

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Ideological systems and its validation: a neutrosophic approach

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Abstract. Ideologies face two critical problems in the reality, the problem of commitment and the problem of validation. Commitment and validation are two separate phenomena, in spite of the near universal myth that the human is committed because his beliefs are valid. Ideologies not only seem external and valid but also worth whatever discomforts believing entails. In this paper the authors develop a theory of social commitment and social validation using concepts of validation of neutrosophic logic.

Keywords: Commitment, Ideology, Neutrosophic logic, Social systems, Superstructure, Validation.

1 INTRODUCTION

Ideologies are systems of abstract thought applied to public matters and thus make this concept central to politics. *Ideology* is not the same thing as Philosophy. Philosophy is a way of living life, while ideology is an almost ideal way of life for society. Some attribute to ideology positive characteristics like vigor and fervor, or negative features like excessive certitude and fundamentalist rigor. The word *ideology* is most often found in political discourse; there are many different kinds of ideology: political, social, epistemic, ethical, and so on.

Karl Marx [1] proposes an economic base superstructure model of society (See Figure 1). The *base* refers to the means of production of society. The *superstructure* is formed on top of the base, and comprises that society's ideology, as well as its legal system, political system, and religions. For Marx, the base determines the superstructure. Because the ruling class controls the society's means of production, the superstructure of society, including its ideology, will be determined according to what is in the ruling class's best interests. Therefore the ideology of a society is of enormous importance since it confuses the alienated groups and can create false consciousness.

Minar [2] describes six different ways in which the word "ideology" has been used:

1. As a collection of certain ideas with certain kinds of *content*, usually normative;
2. As the *form or internal logical structure* that ideas have within a set;
3. By the role in which ideas play in *human-social interaction*;
4. By the role that ideas play in the *structure of an organization*;
5. As meaning, whose purpose is *persuasion*; and
6. As the *locus* of social interaction, possibly.

Althusser [3] proposed a materialistic conception of ideology. A number of propositions, which are never untrue, suggest a number of other propositions, which are, in this way, the essence of the lacunar discourse is what is *not* told (but is suggested). For example, the statement *all are equal before the law*, which is a theoretical groundwork of current legal systems, suggests that all people may be of equal worth or have equal opportunities. This is not true, for the concept of private property over the means of production results in some people being able to own more than others, and their property brings power and influence. Marxism itself is frequently described as ideology, in the sense in which a negative connotation is attached to the word; that is, that Marxism

is a *closed system* of ideas which maintains itself in the face of contrary experience. Any social view must contain an element of ideology, since an entirely objective and supra-historical view of the world is unattainable. Further, by its very scope and strength, Marxism lends itself to transformation into a closed and self-justifying system of assertions.

For Mullins [4], an ideology is composed of four basic characteristics:

1. It must have power over cognitions;
2. It must be capable of guiding one's evaluations;
3. It must provide guidance towards action;
4. And, as stated above, must be logically coherent.

Mullins emphasizes that an ideology should be contrasted with the related (but different) issues of *utopia* and *historical myth*. For Zvi Lamm [5] an ideology is a system of assumptions with which people identify. These assumptions organize, direct and sustain people's volitional and purposive behaviour. The assumptions on which an ideology is based are not collected at random but constitute an organized and systematic structure. An ideology is a belief system which explains the nature of the world and man's place in it. It explains the nature of man and the derivative relationships of humans to one another.

Mi Park [6] writes, "*Ideology is the main medium with which conscious human beings frame and re-frame their lived experience. Accumulated memories and experiences of struggle, success and failure in the past influence one's choice of ideological frame"*. In according to Cranston [7] an ideology is a form of social or political philosophy in which practical elements are as prominent as theoretical ones. A system of ideas aspires both to explain the world and to change it. Therefore, the main purpose behind an ideology is to offer change in society through a normative thought process. For Duncker [8] the term ideology is defined in terms of a system of presentations that explicitly or implicitly claim to absolute truth.

Ideas may be good, true, or beautiful in some context of meaning but their goodness, truth, or beauty is not sufficient explanation for its existence, sharedness, or perpetuation through time. *Ideology* is the ground and texture of cultural consensus. In its narrowest sense, this may be a consensus of a marginal or maverick group. In the broad sense in which we use the term ideology is the system of interlinked ideas, symbols, and beliefs by which any culture seeks to justify and perpetuate itself; the web of rhetoric, ritual, and assumption through which society coerces, persuades, and coheres. Therefore:

1) An Ideology is a system of related ideas (learned and shared) related to each other, which has some permanence, and to which individuals and/or human groups exhibit some commitment.
2) Ideology is a system of concepts and views, which serves to make sense of the world while obscuring the social interest that are expressed therein, and by completeness and relative internal consistency tends to form a closed belief system and maintain itself in the face of contradictory or inconsistent experience.
3) All ideology has the function of constituting concrete individuals as subjects (Althusser, [3]).

Conventional conceptions of author (authority, originator) and individual agent are replaced by the ideologically constituted actor subject. Stereotypes, that actor subject rely on to understand and respond to events. As much if the Philosophy, Political or Religion is doxical reflected of economic relations as if they express in a specific language certain mental model of human relations, or an update of a certain field of a common structure to society, only be closed the debate after a theoretical treatment.

Nevertheless, theoretical treatment of all ideology firstly has to be located to synchronism level. Relation between synchronous and diachronic order is complicated when we are located in a unique level: the structure of a social system and transformations are homogenous among them. In the case of synchrony are constructed static or dynamic models. In the diachronic case we will have to consider History, content multiform movement making take part heterogeneous elements. Ideology emerges spontaneously at every level of society, and simply expresses the existing structure of the Social System. Members of every class construct their own understanding of the social system, based on their personal experiences. Since those experiences are primarily of capitalist social relations, their ideology tends to reflect the norms of capitalist society. The individual subject is faced, not with the problem of differentiating the *ideological* from the *real*, but with the problem of choosing between competing ideological versions of the *real*. Drawing on Jaques Lacan's theory in which human subjectivity is formed through a process of misrecognition of the ideology in the *mirror* of language.

This is far from the only theory of economics to be raised to ideology status - some notable economically-based ideologies include *mercantilism, mixed economy, social Darwinism, communism, laissez-faire economics, free trade, ecologism, Islamic fundamentalism*, etc. *Science* is an ideology in itself. Therefore, while the scientific method is itself an ideology, as it is a collection of ideas, there is nothing particularly wrong or bad about it. In everything what affects the study of the ideologies the problem has a double sense:

1) *Homogeneity:* each discourse informs a content previously given and that puts under its own syntaxes.

2) *Heterogeneity:* passage of the reality to languages introduces a complete displacement of all the notions, fact that excludes the cause that they are conceived like simple duplicates.

In Deontical Impure Systems (DIS)[1] approach, the Superstructure of Social System has been divided in two ([9-20]):

1) *Doxical Superstructure (DS)* is formed by values in fact, political and religious ideologies and culture of a human society in a certain historical time.

2) *Mythical Superstructure (MS)* also has been divides in two parts:

   a) MS_1 containing the mythical components or primigenial bases of the ideologies and cultures with the ideal values.

   b) MS_2 containing ideal values and utopias that are ideal wished and unattainable goals of belief systems of the Doxical Superstructure (DS).

It is summarized these ideas in the following diagram (Figure 1):

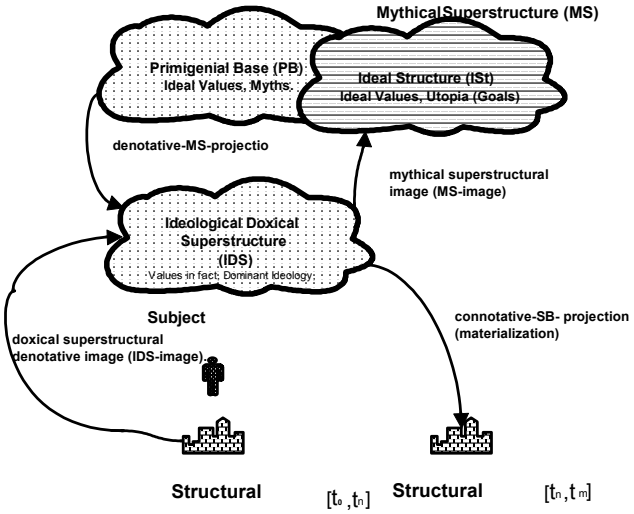

Figure 1: Deontical Impure Systems (DIS) approach

The following elements ([20-21]) are listed in the order that would be logically required for the understanding a first approach of an ideology. This does not imply priority in value or in causal or historical sense.

1) *Values*. Implicitly or explicitly, ideologies define what is good or valuable. We refer to ideal values belonging to Mythical Superstructure (MS). They are goals in the sense that they are the values in terms of which values in fact belonging to Doxical Superstructure (DS) are justified. Ideal values tend to be abstract summaries of the behavioral attributes which social system rewards, formulated after the fact. Social groups think of themselves, however, as setting out to various things in order to implement their values. Values are perceived as a priori, when they are in fact a posteriori to action. Having abstracted a ideal value from social experience in SB, a social group may then reverse the process by deriving a new course of action from the principle. At the collective level of social structure (SB), this is analogous to the capacity for abstract thought in individual subjects and allows great (or not) flexibility in adapting to events. Concrete ideologies often substitute observable social events for the immeasurable abstract ideal values to give the values in fact immediate social utility.

[1] *Impure sets* are sets whose referential elements (absolute beings) are not counted as abstract objects and have the following conditions: a) They are real (material or energetic absolute beings). b) They exist independently of the Subject. c) S develops p-significances on them. d) True things can be said about them. e) Subject can know these true things about them. f) They have properties that support a robust notion of mathematical truth. A *simple impure system-linkage* $\Sigma\equiv$ (M, R) is a semiotic system consisting of the pair formed by an impure object set M the elements of which are p-significances (relative beings) of entities belonging to Reality (absolute beings) or certain attributes of these, and a set of binary relations, such that $R \subset P(M \times M) = P(M^2)$. That is $\forall r \in R/r \subset M$ XM being $r = \left\{ \left(x_i, y_j \right) \in MxM / x_i, y_j \in M \right\}$. An *impure system-linkage* defined within an impure object set M is a simple system S = (M, R) or a finite union of simple systems-linkage $\Sigma = \cup^{n}_{i=1} \Sigma_i$ such that Σ_i are simple systems. This shall be denoted as $\Sigma \equiv$ (M, R) such that $R \subset P(\cup_{finite} M^2)$. A *Deontical system* is an organization of knowledge on the part of the subject S that fulfils the following ones: a) Other subjects (human beings) are elements of the system. b) Some existing relations between elements have Deontic modalities. c) There is purpose (purposes) ([9-17], [20]).

2) Substantive beliefs (Sb) [2]. They are the more important and basic beliefs of an ideology. Statements such as: *all the power for the people, God exists, Black is Beautiful*, and so on, comprise the actual content of the ideologies and may take almost any form. For the believers, substantive beliefs are the focus of interest.

3) Orientation. The believer may assume the existence of a framework of assumptions around his thought, it may not actually exist. The orientation he shares with other believers may be illusory. For example, consider almost any politic and sociologic ideology. Such system evolves highly detailed and highly systematic doctrines long after they come into existence and that they came into existence of rather specific substantive beliefs. The believers interact, share specific consensuses, and give themselves a specific name: Marxism, socialism, Nazism, etc. Then, professionals of this ideology work out an orientation, logic, sets of criteria of validity, and so forth.

4) Language. It is the logic of an ideology. Language **L** of an ideology is the logical rules which relates one substantive[2] belief ([12-17], [20]) to another within the ideology. Language must be inferred from regularities in the way of a set of substantives beliefs in the way a set of beliefs is used. The language will be implicit, and it may not be consistently applied. Let Sb be a substantive belief. We propose the following rules of generation of ideologies:

$$R1 \to \Pr ed \to subs\tan tive \quad beliefs \to \bigcup_{k=1}^{n} Sb_k = Sb_j$$

$$R2 \to Arg \to hypothesis + goal \to why \quad and \quad what \quad for?$$

$$R3 \to T \to \Pr ed \wedge Arg$$

$$R4 \to T \to T\left[(\&)\right]^n, n \geq \varnothing$$

$$R5 \to \& = \begin{cases} \wedge \\ \vee \\ \equiv \end{cases}$$

Argument is formed by the sum of two characteristics: hypothesis, that is to say, so that this physical and social reality? And goal: as we want is this society to reach its "perfection" (utopia).

5) *Perspective.* Perspective of an ideology or their cognitive map, is the set of conceptual tools. Central in most perspectives is some statement of where the ideology and/or social group that carries it stands in relation to other things, specially nature, social events or other social groups. *Are we equals? Enemies? Rulers? Friends?* Perspective as description of the social environment is a description of the social group itself, and the place of each individual in it. The perspective may be stated as a myth in the Mythical Superstructure ([16-17], [20]). It explains not only who subjects are and how subjects came to be in cognitive terms, but also why subject exist in terms of ideal values. Meaning (d-significances $s_{\Sigma}^{\overrightarrow{D}}$)[3] and identification are provided along with cognitive orientation.

6) Prescriptions and proscriptions. This includes action alternatives or policy recommendations as well as deontical norms for behavior. They are the connotative-SB-projection from IDS to SB (see figure 1). Historical examples of prescriptions are the Marx's *Communist Manifesto,* the Lenin's *What is To Be Done* or the Hitler's *Mein Kampf.* Deontical norms represent the cleanest connection through of MS-image and SB-projections between the abstract idea (in Ideal Structure belonging to Mythical Superstructure) and the concrete applied belief because they refer to behavior that is observable. They are the most responsive conditions in being directly carried by the social group through the mechanisms of social reward and punishment.

[2] Substantive beliefs ([14-15], [20]) constitute the axioms of the system, while many of derived beliefs will constitute their theorems.

[3] Denotation (d-s) is the literal, obvious definition or the common sense of the significance of a sign. We denote s to the systemic significance being a denotative significance. ζ is the set of significant (signs) of Reality and ζ_{Σ} to the set of systemic significants, e.g. the part of signs that have been limited by the Subject when establishing the borders of the system, and so that $\zeta_{\Sigma} \subset \zeta$. Denotative systemic significance (d-s) s_{Σ} *is a function defined in* ξ *so that if* $\xi_{\Sigma} \subset \xi$ *then* $s_{\Sigma}\left(\xi_{\Sigma}\right) \subseteq \xi_{\Sigma}$ .Denotative systemic significance (d-significance) is the significance of the absolute beings. Denotative systemic significance (d-significance) agrees with relative beings.

7) *Ideological Technology*. In according Borhek and Curtis (1983) every ideology contains associated beliefs concerning means to attain ideal values. Some such associated beliefs concern the subjective legitimacy or appropriateness of d-significances, while others concern only the effectiveness of various d-significances. For example, political activists and organizational strategy and tactics are properly called technology of the ideology. Ideological Technology is the associated beliefs and material tools providing means for the immediate (in Structural Base) or far (In Ideal Structure as Utopia) goals of an ideology. Ideological Technology is not used to justify or validated other elements of an ideology, although the existence of ideological technologies may limit alternative among substantive beliefs. Ideological Technology commands less commitment from believers than do the other elements. A change in Ideological Technology (*strategy*) may be responsible for changes in logical prior elements of an ideology. Ideological Technology, like belonging to Structural Base and having a series of prescriptions concerning *doing* can influence the life conditions of believers, thus forcing an adaptation in the ideology itself. Eurocomunism in Western Europe gives to a good historical example. Ideological Technology may become symbolic through DS-image and an inverse MS-image on Primigenial Base belonging to Mythical Superstructure, and it can cause of more fundamental differences between ideologies and, therefore, a source of conflict. Conflicts between anarchists and Communists in the Spanish Civil War or the ideas of Trotsky and those of Stalin in the USSR are examples of it. Much blood has been shed between Muslims and Hindus over the fact that their religions have different dietary restrictions (deontical prohibitions).

Then:

1) Conflicts are not over Ideological Technology but over what technological difference symbolizes in the Primigenial Base of the Mythical Superstructure.
2) Substantive beliefs are understood only in terms of ideal values, criteria of validity, language and perspective.
3) The believer is usually better able to verbalize substantive beliefs than he is values, criteria, logical principles or orientation, which is apt to be the unquestioned bases from which he proceeds.
4) Ideal values, criteria of validity, language and perspective may have been built up around a substantive belief to give it significance and justification.

Based on these criteria and our DIS approach, we are able to propose the following definition of ideology:

Definition 1 *We define systemically as* ideology *and we represent as* $Id = \langle Sb, IR \rangle$ *to the system formed by an object set Sb whose elements are substantive beliefs* $Sb = \{Sb_i\}, i = 1,...,n$ *and whose relational ser IR is formed by the set of binary logical abstract relations between substantive beliefs.*

2 VALIDATION OF IDEOLOGIES

Ideologies face two critical problems in the reality, the problem of commitment and the problem of validation. Ideologies persist because they and/or the social vehicle that carry them are able to generate and maintain commitment. For commitment to be maintained, however, an ideology must also, independently, seem to valid. Commitment and validation are two separate phenomena, in spite of the near universal myth that the human is committed because his beliefs are valid. Ideologies not only seem external and valid but also worth whatever discomforts believing entails. Humans often take the trouble to validate their beliefs because they are committed to them. An ideology with high utility limits available alternative ideology by excluding them, and limitation of alternatives increases the utility of whatever one has left. Utility for a group is not always identical to individual utility that motivates group reinforcement. Insofar as humans must collaborate to attain specific goals, they must compromise with collective utilities. Groups retain or change ideologies according to the history of reinforcement.

By virtue of its structure (within the Doxical Superstructure DS), an ideology may be able to fend off negative evidence in a given stimuli social environment H' but experience difficulty as social conditions (within Structural Base SB) change (See figure 1). Ideologies may respond to a changing social environment not only with adjustments in the social vehicles that carry those (Social States), but also with changes in the ideological logic (Semiotic States). Consider the possibilities that are open when an ideology is challenged by stimuli:

1) The ideology may be discarded, or at least the level commitment reduced.

2) The ideology may be affirmed in the very teeth of stimuli (*the triumph of faith*).

3) The believers may deny that the stimuli (events) were relevant to the ideology, or that the substantive belief that was changelled was importantly related to the rest of ideology.

The validation of belief is a largely social process. The social power of ideology depends on its external quality. Ideologies seem, to believers, to transcend the social groups that carry them, to have an independent existence of their own ([21-22]). For ideologies to persist must not only motivate commitment through collective utility but also through making the ideology itself seem to be valid in its own right. Perceived consensus is a necessary but not sufficient condition for the social power of ideologies. Therefore ideological validation is not simply a matter of organizational devices for the maintenance of believer commitment, but also of the social arrangements wherever the abstract system of ideology is accorded validity in terms of its own criteria. The appropriate criteria for determining validity or invalidity are socially defined. Logic and proofs are just as much social products as the ideologies they validate.

Cyclical principle of validation: *An idea is valid if it objectively passes the criterion of validity itself.*

Conditions of validation:

1) *Social condition*: Criterion of validity is chosen consensually and it is applied through a series of social conventions (Berger and Luckmann, 1966).

2) *First nonsocial condition*: Ideology has a logic of its own, which may not lead where powerful members of the social group wanted to.

3) *Second nonsocial condition*: The pressure of events (physical or semiotic stimuli coming from the stimulus social environment H') that may be pressure on believers to relinquish an ideology. For an ideology to survive the pressure of events with enough member commitment to make it powerful it must receive validation beyond the level of more consensuses.

The pressure of the events is translated in form denotative significances as DS-images on the component subjects of the Dogmatic System of the set of believers belonging to Structural Base.

Main Principle of validation: *The power of an ideology depends on its ability to validate itself in the face of reason for doubt.*

The *internal evidence* of an ideology (IE) is the data which derive from the ideology itself or from a social group or organization to which is attached. For highly systematic belief system (an ideology), any attack upon any of its principles is an attack upon the system itself. Then:

1) If one of the basic propositions (substantive beliefs) of an ideology is brought under attack, then so the entire ideology. In consequence, an ideology is at the mercy of its weakest elements.

2) An ideology has powerful conceptual properties, but those very properties highlight the smallest disagreement and give it importance in its logical connections with other items of ideology.

3) Even if an ideology is entirely nonempirical, it is vulnerable because even one shaken belief can lead to the loss of commitment to the entire ideological structure.

4) An ideology as the religious ideologies, with relatively little reference to the empirical world cannot be much affected by external empirical relevance, simply because the events do not bear upon it. The essential substantive belief in the mercy of God can scarcely be challenged by the continued wretchedness of life.

5) Nevertheless, concrete ideologies are directly subject to both internal and external evidence.

6) The abstract ideology is protected from external evidence by its very nature. A cult under fire may be able to preserve its ideology only by retreating to abstraction. Negative external evidence may motivate system-building at the level of the abstract ideology, where internal evidence is far more important.

7) The separibility of the abstract ideology from its concrete expression depends on the ability of believers no affiliated with the association (cult and/or concern) that carried it socially to understand and use it, that is to say, subjects belonging to the Structural Base.

8) If the validation of an ideology comes from empirical events and the ability to systematically relate propositions according to an internally consistent logic, it can be reconstructed and perpetual by any social group with only a few hints.

9) The adaptation of an ideology is some sort of compromise between the need of *consensual*

validation and the need for independence from the associations that carries it.

Consensual validation is the confirmation of reality by comparison of one's own perceptions and concerns with those of others, including the recognition and modification of distortions. Consensual validation, describes the process by which human being realize that their perceptions of the world are shared by others. This bolsters their self-confidence since the confirmation of their observations normalizes their experience. Consensual validation also applies to our meanings and definitions. Arriving at a consensus of what things mean facilitates communication and understanding. When we all agree what something is, the definition of that something has integrity. Reality is a matter of consensual validation ([23]). Our exact internal interpretations of all objects may differ somewhat, but we agree on the generic class enough to communicate meaningfully with each other. Phantasy can be, and often is, as real as the "*real world*." Reality is distorted by strong, conflicted needs. People seek affiliations with groups that enable them to maintain an ideal balance between the desires to fit in and stand out. These motives operate in dialectical opposition to each other, such that meeting one signal a deficit in the other and instigate increased efforts to reduce this deficit. Thus, whereas feelings of belonging instigate attempts to individuate one, feelings of uniqueness instigate attempts to re-embed oneself in the collective. The physicalistic accretion to this rule of consensual validation is that, physical data being the only "real" data, internal phenomena must be reduced to physiological or behavioural data to become reliable or they will be ignored entirely. Public observation, then, always refers to a limited, specially trained public. It is only by basic agreement among those specially trained people that data become accepted as a foundation for the development of a science. That laymen cannot replicate the observations is of little relevance. What is so deceptive about the state of mind of the members of a society is the "consensual validation" of their concepts. It is naively assumed that the fact that the majority of people share certain ideas and feelings proves the validity of these ideas and feelings. Nothing is further from the truth.

Consensual validation as such has no bearing whatsoever on human reason. Just as there is a "*folie a deux*" there is a "*folie a millions*." The fact that millions of people share the same vices does not make these vices virtues and the fact that they share so many errors does not make the errors to be truths ([25]). On the other hand, when the ideology is identified with the community (or with a consensus), and this community, as well, it is not truly identified with a true socio-political institution based on the land (nation), but with a transcendental principle, personified in the norms of a church, sect or another type of messianic organization, its effects on the secular political body, within as it prospers but with which it is not identified, they are inevitable and predictable destructive. The process of consensual validation, then ties the content of ideological beliefs to the social order (existing in the Structural Base) itself. It is established a feedback process: 1) If the social order remains, then the ideological beliefs must somehow be valid, regardless of the pressure of the events. 2) If the ideological beliefs are agreed upon by all, then the social order is safe.

Commitment of believers is the resultant of two opposite forces.

1) Social support (associations and no militant people), which maintains ideology.

2) Problems posed by pressure of events, which threaten ideology.

When ideology is shaken, further evidence of consensus is required. This can provide by social rituals of various sorts, which may have any manifest content, but which act to convey the additional messages ([23]). Each member of a believer group, in publicly himself through ritual is rewarded by the public commitment of the others. Patriotic ceremonies, political meetings, manifestations by the streets of the cities, transfers and public religious ceremonies are classic examples of this. Such ceremonies typically involve a formal restatement of the ideal ideology in speeches, as well as rituals that give opportunities for individual reaffirmation of commitment. For Durkheim ([21]) ideological behaviour could be rendered sociologically intelligible by assuming an identity between societies and the object of worship. The ideal of all totalitarian ideology is the total identity between the civil society and the ideological thought, that is to say, the establishment of the unique thought without fissures. Thus consensual validation and validation according to abstract ideal (Ideal Mythical Superstructure) are indistinguishable in the extreme case. If a certain ideology has a sole raison d'être affirmation of group membership (fundamentalist ideologies), no amount of logical or empirical proof is even relevant to validation, though proofs may in fact be emphasized as part of the ritual of group life.

We have the following examples of consensual validation in actual ideologies. :

1) F*alse patriotism* is the belief that whatever government says goes.

2) *Neo-conservatism* is the belief that the *status quo* should be maintained.

3) *Radical Progressism* is the belief that the social reality can change undermining the foundations of a millenarian culture.

4) *Shallow utilitarianism* is whatever the majority says goes, and since the majority, that's what shallow utilitarian believe in. This is often called *groupthink*. Erich Fromm ([25]) called it "*the pathology of normalcy*" and claimed it was brought about through consensual validation.

5) *Islamic fundamentalism.* From the perspective of the Islamists, his Islamic behaviour makes him a moral person. Living the dictates of Islam makes him "*good*." He does well, and he is good. His ethical beliefs and actions find consensual validation and continuous reinforcement in any and every geographical area of the *umma*. He no longer doubts, no longer even wonders. In a crude sense, he knows who he is, where he belongs, and what his purpose in life is. He knows never to doubt. His is not to reason why. Besides, he has lost the will, if not the capacity. By Islamic standards, the most virulent jihad is good. Jihadism is the ethical life of Islam. The Islamist embraces it right down to the last mitochondrion in the last cell of his body. He could not give up Islam even if he wanted, and he never commits the perditious sin of wanting.

2.1 Neutrosophic logic approach to validation

For a logical approach to the validation of ideologies, we will use the Neutrosophic logic ([20], [26-32]) (See figure 1).

Definition 2: True IDS-image is the *IDS-image which is permitted syntactically and semantically and whose external evidence provides with a degree of truth value in its existence.*

Considering the neutrosophic principles we shall establish the following Axioms:

Axiom 1: *Any IDS-image IDSi is provided with a neutrosophic truth value* $\wp$*, element of a neutrosophic set E =]* $^{-}0, 1^{+}[^{3}$*. non enumerable and stable for multiplication.*

Axiom 2: *Any IDS-image IDSi is provided with a neutrosophic veritative value* $v \in \left]^{-}0,1^{+}\right[^{3}$ *such that v =* $V(\wp)=V((T, I, F))$*, V reciprocal application of E in* $\left]^{-}0,1^{+}\right[^{3}$ *and which possesses the following properties:*

1) $V(0) = {}^{-}0$.

2) $V(\wp_1, \wp_2) = V(\wp_1).\ V(\wp_2)$.

If $T = 1^{+}$ it will designate absolute truth and if $T= {}^{-}0 \equiv F = 1^{+}$ it will designate the absolute falseness of the IDS-image. If complementariness is designated by M, the principle of complementariness between two IDs-images: IDSi and $\widehat{IDSi}$, it there is iff $(\wp_1 + \wp_2) \in \left]^{-}0,1^{+}\right[^{3}$. When $\wp_1 \neq 0$ y $\wp_2 \neq 0$, such that v ($\wp_i^k$ M $IDSi_{kj}$) = 0, it is necessary that $\wp_1 + \wp_2 = {}^{-}0$, as the sum of veracities does not admit opposing elements.

Axiom 3: *If* $\neg IDSi$ *designates the non-IDS-image* $IDSi$*, with the neutrosophic truth value* $\neg\wp$*, we will have to* $V(\wp + \neg\wp) = 1^{+}$.

Axiom 4: $\forall IDSi \in L$ / $v = V(\wp) = V((T, I, F)) = ((1^{+}, {}^{-}0, {}^{-}0))$

Definition 3: Absolute true IDS-image TIDSi *is the IDS-image that fulfill* $v = V(\wp) = V((T, I, F)) = ((1^{+}, {}^{-}0, {}^{-}0))$

Let S be a Believer Subject. Let IDSi be a IDS-image. We denote as Δ the operator *a priori and the equivalence operator as* $\equiv$. We shall designate as $(\Delta\equiv)$ the *equivalence a priori* operator and as $(\Box\equiv)$ the necessarily *equivalent operator.* We shall designate as V the *true being* operator and as $\Box$V the necessarily *true* operator. We designate as F the *false being* operator. We shall designate the *equivalent a posteriori* operator as $\nabla\equiv$. We may establish the following Theorems:

Theorem 1: *Each absolute true IDS-image IDSi considered by S is equivalent a priori to a necessary IDs-image* $*IDSi$*, that is,* $\forall IDSi \exists *IDSi\ (IDSi(\Delta\equiv)\Box *IDSi)$.

Proof:

We shall consider the neutrosophic veritative value $v \in \left]^{-}0,1^{+}\right[^{3}$ of a specific IDS-image IDSi which shall be $T=1^{+}$ if it is true and $T= {}^{-}0 \equiv F = 1^{+}$ if it is false. Therefore $IDSi \leftrightarrow T = 1^{+}$ is a priori by stipulation, and $T = 1^{+}$ is necessary if IDSi is true and necessarily

false F = 1^+ if r_{ij} is false. That is $IDSi(\Delta \equiv) * \wp_i^k$ and $(\Box * IDS_i \vee \Box \neg * IDSi)$.

Theorem 2: *Each absolute true IDS-image IDSi considered by S is necessarily equivalent to an a priori S-image* $*IDSi$: $\forall IDSi \exists * IDSi$ ($r_{ij} IDSi$ $(\Box \equiv)$ $\Delta * IDSi$).

Proof:

a) Given a true IDS-image IDSi we establish the IDS-image ${}^A IDSi$ for T = 1^+ and such that $IDSi(\Delta \equiv)^A IDSi$ VIDSi $\Rightarrow \Box V\ {}^A IDSi$. If IDSi has as neutrosophic veritative value T = 1^+, then $IDSi \equiv {}^A IDSi$ will have the same neutrosophic veritative value, therefore IDSi($\Box \equiv$) $IDSi \equiv {}^A IDSi$. Thus, we have demonstrated that each true IDS-image is necessarily equivalent IDS-image a priori, specifically $IDSi \equiv {}^A IDSi$.

b) In the case of F r_{ij}, the existence of a IDS-image $*IDSi$ will be necessary such that (IDSi($\Box \equiv$) $*IDSi \wedge \Delta \neg * IDSi$). For FIDSi $\Rightarrow$ F ${}^A IDSi$ as ($IDSi \equiv {}^A IDSi$)($\Box \equiv$) $\neg IDSi$ and therefore, due to this selection of $*IDSi$ there cannot be (IDSi ($\Box \equiv$) $*IDSi$. For FIDSi, $*IDSi$ is chosen, that is $\neg$ ($IDSi \equiv {}^A IDSi$). Thus, clearly there is (IDSi ($\Box \equiv$) $*IDSi \wedge \Delta \neg * IDSi$) due to the selection of $*IDSi$. Therefore, Theorem 2 is demonstrated.

Theorem 3: *Each necessary IDS-image IDSi considered by S is equivalent a posteriori to an absolute true IDS-image* $*IDSi$ *, that is,* $\forall IDSi \exists * IDSi$ ($\Box IDSi(\nabla \equiv) * IDSi$).

Proof:

If IDSi V $(\wp) = v = (1^+, {}^-0, {}^-0)$ is necessary.

$\Box$

$IDSi \Rightarrow (\nabla \equiv)\left(*IDSi / v = (1^+, {}^-0, {}^-0) \vee *IDSi / v = ({}^-0, {}^-0, 1^+)\right)$. The second term $*r_{ij} / v = ({}^-0, {}^-0, 1^+)$ implies F $*r_{ij}$ which contradicts Theorem 1 as the IDS-image IDSi is true, being equivalent a priori to $*IDSi$ which is necessary and therefore $v = (1^+, {}^-0, {}^-0)$.

Theorem 4: *Each a posteriori IDS-image IDSi considered by S is necessarily equivalent to an absolute true IDS-image* $*IDSi$: $\forall IDi \exists * IDSi$ ($\nabla IDSi (\Box \equiv)$ $*IDSi$).

Proof.

If $*IDSi$ has $v = (1^+, {}^-0, {}^-0)$ as being true, it will imply that $\left(IDSi / v = (1^+, {}^-0, {}^-0) \vee IDSi / v = ({}^-0, {}^-0, 1^+)\right)$. For $v = (1^+, {}^-0, {}^-0)$ it is obvious. For $v = ({}^-0, {}^-0, 1^+)$ it contradicts Theorem 2.

3 VARIABLES OF AN IDEOLOGY

Ideologies "are" in the Superstructure, but far from our intention to think about neoplatonic ideas that beliefs exist *per se*, without material support. Without believers there is no belief system; but the belief system itself is not coextensive with any given individual Subject or set of Subjects. Ideologies as belief system have longer lives than Subjects and are capable of such complexity that they would exceed the capacity of a given Subject to detail. Ideologies have the quality of being real and having strong consequences but having no specific location, because Superstructure has not a physical place. In according to Rokeach ([33]), people make their inner feelings become real for others by expressing them in such cases as votes, statements, etc. they built or tear dhow, which in turn form the basis of cooperative (or uncooperative) activity for humans, the result of which is "Reality". Ideology is one kind of Reality although not all of it. Ideologies, like units of energy (information), should be thought of as things which have variable, abstract characteristics, not as members of platonic categories based on similarity. The ideological variables are:

1) Interrelatedness of their substantive beliefs defines the *degree of an ideology* (DId) and it is defined like the number m of their logical abstract relations. Logically, some belief systems ideologies are more tightly interrelated than others. We suppose the ideologies and belief sys-

tems forming a continuum: $[Id_l,...,Id_r]$. Then: a) At the right end of the continuum are ideologies that consist of a few highly linked general statements from which a fairly large number of specific propositions can be derived. Confronted by a new situation, the believer may refer to the general rule to determine the stance he should take. Science considered as ideology is an example. b) At the left end of the continuum are ideologies that consists of sets of rather specific prescriptions and proscriptions (deontical norms) between which there are only weak functional links, although they may be loosely based on one or more assumptions. Confronted by a new situation, the believer receives little guidance from the belief system because there are no general rules to apply, only specific behavioral deontical norms that may not be relevant to the problem at hand. Agrarian religions are typically of this type. They are not true ideologies but proto-ideologies. If DId is defined by m or number of logical abstract relations between substantive beliefs, then m = 0 defines the non existence of belief system and $m = \infty$ an ideal ideology that it contemplated understanding of the totality, that is to say, of the own Reality. Consequences: a) High DId may inhibit diffusion. It may make an otherwise useful trait inaccessible or too costly by virtue of baggage that must accompany it. Scientific theories are understood by a small number of experts. b) To DId is high, social control may be affected on the basis of sanctions and may be taught and learned. Ideologies with a relative high DId seem to rely on rather general internalized deontical norms to maintain social control.

2) The *empirical relevance* (ER) is the degree to which individual substantive belief Sb_i confront the empirical world (Reality). The proposition that the velocity is the space crossed by a mobile divided by the time that takes in crossing that space has high empirical relevance. The proposition God's existence has low empirical relevance. $ER \subset [0,1]$, being 0 null empirical relevance (*Homo neaderthalensis* lives at the moment) and 1 total empirical relevance (a + b = c). When beliefs lacking empirical relevance arise in response to pressing strain in the economic or political structures (SB), collective action to solve economic or political problems becomes unlikely. Lack of ER protects the ideology and the social vehicle from controversies arising between the highly differentiated populations of believers.
3) The *ideological function* is the actual utility for a group of believing subjects. Ideological function conditions the persistence of the ideology, or time that is useful or influences social structure.
4) The *degree of the willingness* of an ideology (WD) is the degree to which an ideology accepts or rejects innovations. $WD \subset [0,1]$ being WD = 0 null acceptance and WD = 1 total acceptance. To major consequence of WD to take innovations is the ease with which ideologies adapt changes in their social environment. Beliefs with $WD \approx 1$, accepting innovations of all ideological degrees survive extreme changes in social structure: Shinto in Japan or Roman Catholicism is examples.
5) The *degree of tolerance* of an ideology (TD) is the degree with an ideology accepts or rejects competing ideologies or beliefs systems. $TD \subset [0,1]$ being TD = 0 total rejection and TD = 1 total acceptance. Some accepts *all* others as equally valid but simply different explanations of reality $TD \approx 1$. Others reject all other ideology as evil $TD \approx 0$, and maintain a position such as one found in revolutionary or fundamentalist movements. Then: a) High TD seems to be independent of ideological system and the degree of the willingness (WD). b) Low TD is fairly strong related with WD. c) Low TD is fairly strong related with a high ER. Relevance of highly empirical beliefs to each other is so clear. Therefore $TD = f\left(\frac{1}{WD}, ER\right)$. TD has consequences for the ideology: 1) It affects the case with the organizational vehicle (social structure) may take alignments with other social structures. 2) It affects the social relationships of the believers.
6) The *degree of commitment* demanded by an ideology (DCD) is the intensity of commitment demanded to the believer by the part of the ideology or the type of social vehicle by which the ideology is carried. $DCD \subset [0,1]$ being DCD = 0 null commitment demanded and DCD = 1 total adhesion. Then: a) DCD is not dependent of ideological system ID, empirical relevance (ER), acceptance or innovation (WD) and tolerance (TD). b) The degree of commitment demanded (DCD) has consequences for the persistence of the ideology. If an ideology has $DCD \approx 1$ and cannot motivate the believers to make this commitment, it is not likely to persist

for very long. Intentional communities having like immediate objective utopias have typically failed in large part for this reason. Revolutionary and fundamentalist ideologies typically demand DCD = 1 of their believers and typically institute procedures, such as party names to both ensure and symbolize that commitment (Crossman, [34]). c) DCD depends of invalidation. Ideological systems with low DCD fail or are invalidated slowly as beliefs drops from the believers' repertoire one by one or are relegated to some inactive status. Invalidation of ideological systems with high DCD produces apostates. High DCD ideological systems seem to become invalidated in a painful explosion for their believers, and such ideologies are replaced by an equally high DCD to an ideology opposing the original one. But reality is not constructed. Reality is encountered and then modified. Human Subjects do, in fact, encounter each other in pairs or groups in situations that require them to interact and to develop beliefs and ideologies in the process. They do so, however, as socialized beings with language, including all its values in fact, logic, prescriptions and proscriptions; in the context of the previous work of others; and constrained by endless social restrictions on alternative courses of action. *Commitment* is focus of ideologies, because is focus Ideas may be good, true, or beautiful in some context of meaning but their goodness, truth, or beauty is not sufficient explanation for its existence, sharedness, or perpetuation through time. *Ideology* is the ground and texture of cultural consensus. In its narrowest sense, this may be a consensus of a marginal or maverick group. In the broad sense in which we use the term ideology is the system of interlinked ideas, symbols, and beliefs by which any culture seeks to justify and perpetuate itself; the web of rhetoric, ritual, and assumption through which society coerces, persuades, and coheres on those aspects of social structure which maintain or create commitment: limitation of alternatives, social isolation, and social insulation through strategies that dictate heavy involvement of the individual Subject in group-centered activities. Individual commitment is view as stemming either from learning and reinforcements for what is learned, or from the fact that ideological functions (actual utility) to maintain personality either by compensating for some feeling of inadequacy, by providing an object for dependence, or by producing order out of disorder (Fromm, 1941; Wallace, 1966). Commitments are validated (or made legitimate) by mechanisms that make them subjectively meaningful to Subjects (Berger and Luckmann, 1966).

7) The *external quality* (EQ) of an ideology ([21]) is the property by which ideologies seem to believers, to transcend the social groups that carry them, to have an independent existence of their own.

Then we propose the following definition:

Definition 3: *Ideological system Id during the time of its actual utility* $[t_0, t_w]$ *or* historical time *is a non-linear function of its main characteristics, such as Id = f(DId, ER, WD, TD, DCD) = f(DId, ER, WD, f'(1/WD, ER), DCD) = F(DId, ER,WD, DCD).*

An ideology varies in the ideological degree (IdD) and its empirical relevance (ER) or the extent to which this ideology pertain directly o empirical reality. The apparent elusiveness of an ideology derives from four characteristics, all of which result from the fact that while beliefs are created and used by humans, they also have properties that are independent of their human use. In according with Borhek and Curtis ([23])

1) Ideologies appear to their believers to have a stability, immutability, coherence and independence. Ideologies to appear to social group members as a suprasocial set of eternal verities, unchangeable thorough mere human action and agreed upon by all right-thinking people not because the verities belong to a believers but because they are true ([21]). In reality, beliefs are changeable.

2) Similarities among substantive beliefs are not necessary parallel structural similarities among ideologies.

3) The historic source of beliefs (the myth) may, by virtue of their original use, endow them with features that remain through millennia of change and particularly fit them to use in novel context.

4) The most important commonality among a set of substantive beliefs is the social structure.

3 CONCLUSIONS

We can draw the following conclusions:

1) Therefore an ideology is a set of beliefs, aims and ideas.

2) Ideologies are not a collection of accidental facts considered separately and referred an underlying history and it is: a) Thoughts about our own behaviors, lives and courses of action. b) A mental impression – something that is abstract in our heads – rather than a concrete thing. c) A system of belief. Just beliefs –non-unchangeable ultimate truths about the way the world should be.
3) Ideology has different meanings:
    1) The process of production of meanings, signs and values in social life.
    2) A body of ideas characteristic of a particular social group or class.
    3) Ideas that help to legitimate a dominant political power.
    4) Socially necessary illusion; the conjecture of discourse and power.
    5) The medium in which conscious social actors make sense of their reality.
    6) Action oriented set of beliefs.
    7) The confusion of linguistic and phenomenal reality.
    8) Semiotic closure.
    9) The indispensable medium in which individuals live out their relations to a social structure.
    10) The confusion of the process whereby social life is converted to a natural.
4) The greater is the ideological degree (Id), the greater is the negative evidence for the whole ideology.
5) The less the degree of empirical relevance, the less the importance of external evidence (pressure of events, but the greater the importance of external evidence.
6) The suprasocial form of an ideology derives most significantly from its abstract ideal form belonging to Mythical Superstructure. The current social influence of an ideology derives of its concrete form belonging to Doxical Superstructure.
7) The more systematic and empirically relevant an ideology is, the greater the feasibility of preserving it as an abstract ideal apart from a given concrete expression.
8) The greater the Ideological degree (DId) and the greater the degree of empiricism, the less the reliance on internal evidence and the greater the reliance of external evidence.
9) The extent of commitment to ideology varies directly with the amount of consensual validation available, and inversely with the pressure of events.

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Modeling and analyzing non-functional requirements interdependencies with neutrosofic logic

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Abstract.

Nonfunctional requirements refer to global properties of software. They are an important part of the requirement engineering process and play a key role in software quality. Current approaches for modelling nonfunctional requirements interdependencies have limitations. In this work we proposed a new method to model interdependencies in nonfunctional requirements using neutrosophic logic. This proposal has many advantages for dealing with indeterminacy making easy the elicitation of knowledge. A case study is shown to demonstrate the applicability of the proposed method.

Keywords: Nonfunctional requirements, requirement engineering neutrosophic logic.

1 Introduction

Software engineers are involved in complex decisions that require multiples points of view. One frequent reason that cause low quality software is associated to problems related to analyse requirements [1]. Nonfunctional requirement (NFR) also known as nonfunctional-concerns [2] refer to global properties and usually to quality of functional requirements. It is generally recognized that NFR are an important and difficult part of the requirement engineering process. They play a key role in software quality, and that is considered a critical problem [3].

The current approach is based solely in modeling interdependencies using only numerical Fuzzy Cognitive Maps (FCM). In this work we propose a new framework for processing uncertainty and indeterminacy in mental models.

This paper is structured as follows: Section 2 reviews some important concepts about Non-functional requirements interdependencies and neutrosofic logic. In Section 3, we present a framework for modelling non-functional requirements interdependencies with neutrosophic logic. Section 4 shows an illustrative example of the proposed model. The paper ends with conclusions and further work recommendations in.

2 Non-functional requirements interdependencies and neutrosofic logic

Nonfunctional requirements are difficult to evaluate particularly because they are subjective, relative and interdependent [4]. In order to analyse NFR, uncertainty arises, making desirable to compute with qualitative information. In software development projects analyst must identify and specify relationships between NFR. Current approaches differentiate three types of relationships: negative (-), positive (+) or null (no contribution). The opportunity to evaluate NFR depends on the type of these relationships.

Softgoal Interdependency Graphs [4] is a technique used for modelling non-functional requirements and interdependencies between them. Bendjenna [2] proposed the use on fuzzy cognitive maps (FCM) relationships between NFCs and the weight of these relationships expressed with fuzzy weights in the range 0 to 1. This model lacks additional techniques for analysing the resulting FCM.

Neutrosophic logic is a generalization of fuzzy logic based on neutrosophy [5]. When indeterminacy is introduced in cognitive mapping it is called Neutrosophic Cognitive Map (NCM) [6]. NCM are based on neutrosophic logic to represent uncertainty and indeterminacy in cognitive maps [5] extending FCM. A NCM is a directed graph in which at least one edge is an indeterminacy one denoted by dotted lines [7]. Building a NCM allows dealing with indeterminacy, making easy the elicitation of interdependencies among NFR.

3 A framework for modelling non-functional requirements interdependencies

The following steps will be used to establish a framework for modeling non-functional requirements interdependencies NCM (Fig. 1).

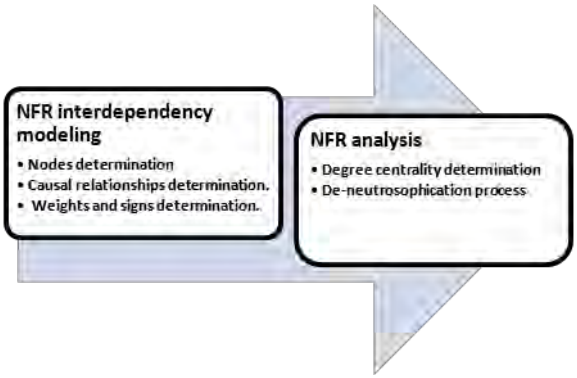

Figure 1: Proposed framework.

- **NFR interdependency modeling**

  The first step is the identification of non-functional concern in a system (nodes). In this framework we propose the approach of Chong based on a catalogue of NFR [4]. Causal relationships, its weights and signs are elicited finally [8].

- **NFR analysis**

  Static analysis is develop to define the importance of NFR based on the degree centrality measure [9]. A de-neutrosophication process gives an interval number for centrality. Finally the nodes are ordered and a global order of NFR is given.

5 Illustrative example

In this section, an illustrative example in order to show the applicability of the proposed model is presented. Five non-functional concerns $R = (NFR_1, \ldots, NFR_5)$ are identified (Table 3).

Table 3 Non-functional requirements

Node	Description
NFR_1	Quality
NFR_2	Reliability
NFR_3	Functionality
NFR_4	Competitiveness
NFR_5	Cost

Table 1 Non-functional requirements

The experts provide the following causal relations (Fig 2) .

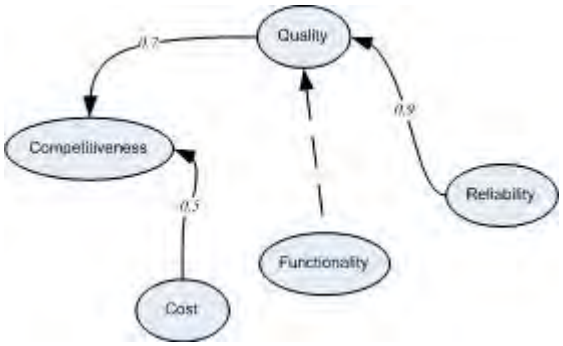

Figure 2. NCM representing NFR interdependencies.

The neutrosophic score of each NFR based on the centralitydegree measure [10] is as follows:

NFR_1 1.6I
NFR_2 0.9
NFR_3 I
NFR_4 1.2
NFR_5 0.5

The next step is the de-neutrosophication process as proposes by Salmeron and Smarandache [11]. $I \in [0,1]$ is repalaced by both maximum and minimum values.

NFR_1 [1.6, 2.6]
NFR_2 0.9
NFR_3 [0, 1]
NFR_4 1.2
NFR_5 0.5

The final we work with extreme values [12] for giving a total order:

$$NFR_1 \succ NFR_4 \succ NFR_2 \succ NFR_3 \sim NFR_5$$

Quality, competitiveness and reliability are the more important concern in this case.

Conclusion

This paper proposes a new framework to model interdependencies in NFR using NCM. Neutrosophic logic is used for representing causal relation among NFR.

Building a NCM allows dealing with indeterminacy, making easy the elicitation of knowledge from experts. An illustrative example showed the applicability of the pro-

posal. Further works will concentrate on two objectives: developing a consensus model and developing an expert system based.

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On Entropy and Similarity Measure of Interval Valued Neutrosophic Sets

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Abstract. In this study, we generalize the similarity measure of intuitionistic fuzzy set, which was defined by Hung and Yang, to interval valued neutrosophic sets. Then we propose an entropy measure for interval valued neutrosophic sets which generalizes the entropy measures defined Wei, Wang and Zhang, for interval valued intuitionistic fuzzy sets.

Keywords: Interval valued neutrosophic set; Entropy; Similarity measure.

1 Introduction

Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities. Smarandache [6] introduced neutrosophic set by adding an inde-terminancy membership on the basis of intuitionistic fuzzy set. Neutrosophic set generalizes the concept of the classic set, fuzzy set [12], interval valued fuzzy set [7], intuitionistic fuzzy set [1], interval valued intuitionistic fuzzy sets [2], etc. A neutrosophic set consider truth-membership, indeterminacy-membership and falsity-membership which are completely independent. Wang et al. [9] introduced single valued neutrosophic sets (SVNS) which is an instance of the neutrosophic set. However, in many applications, the decision information may be provided with intervals, instead of real numbers. Thus interval neutrosophic sets, as a useful generation of neutrosophic set, was introduced by Wang et al. [8]. Interval neutrosophic set described by a truth membership interval, an indeterminacy membership interval and false membership interval.

Many methods have been introduced for measuring the degree of similarity between neutrosophic set. Broumi and Smarandache [3] gave some similarity measures of neutrosophic sets. Ye [11] introduced similarity measures based on the distances of interval neutrosophic sets. Further entropy describes the degree of fuzziness in fuzzy set. Similarity measures and entropy of single valued neutrosophic sets was introduced by Majumder and Samanta [5] for the first time.

The rest of paper is organized as it follows. Some preliminary definitions and notations interval neutrosophic sets in the following section. In section 3, similarity measure between the two interval neutrosophic sets has been introduced. The notation entropy of interval neutrosophic sets has been given in section 4. In section 5 presents our conclusion.

2 Preliminaries

In this section, we give some basic definition re-lated interval valued neutrosophic sets (IVNS) from [8].

Definition 2.1. Let X be a universal set, with generic element of X denoted by x. An interval valued neutrosophic set (IVNS) A in X is characterized by a truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A, with for each $x \in X$, $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

When the universal set X is continuous, an IVNS A can be written as

$$A = \int_X \frac{\langle T_A(x), I_A(x), F_A(x)\rangle}{x}, x \in X.$$

When the universal set X is discrete, an IVNS A can be written as

$$A = \sum_{i=1}^{n} \langle T_A(x_i), I_A(x_i), F_A(x_i)\rangle / x_i \,, x \in X.$$

Definition 2.2. An IVNS A is empty if and only if $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$ and $\inf F_A(x) = \sup F_A(x) = 0$, for all $x \in X$.

Definition 2.3. An IVNS A is contained in the other IVNS B, $A \subseteq B$, if and only if

$$\inf T_A(x) \le \inf T_B(x), \quad \sup T_A(x) \le \sup T_B(x)$$
$$\inf I_A(x) \ge \inf I_B(x), \quad \sup I_A(x) \ge \sup I_B(x)$$
$$\inf F_A(x) \ge \inf F_B(x), \quad \sup F_A(x) \ge \sup F_B(x)$$

for all $x \in X$.

Definition 2.4. The complement of an IVNS A is denoted by A^c and is defined by

$$T_{A^c}(x) = F_A(x)$$
$$\inf I_{A^c}(x) = 1 - \sup I_A(x)$$
$$\sup I_{A^c}(x) = 1 - \inf I_A(x)$$
$$F_{A^c}(x) = T_A(x)$$

for all $x \in X$.

Definition 2.5. Two IVNSs A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

Definition 2.6. The union of two IVNSs A and B is an IVNS C, written as $C = A \cup B$, is defined as follow

$$\inf T_C(x) = \max\{\inf T_A(x), \inf T_B(x)\}$$
$$\sup T_C(x) = \max\{\sup T_A(x), \sup T_B(x)\}$$
$$\inf I_C(x) = \min\{\inf I_A(x), \inf I_B(x)\}$$
$$\sup I_C(x) = \min\{\sup I_A(x), \sup I_B(x)\}$$
$$\inf F_C(x) = \min\{\inf F_A(x), \inf F_B(x)\}$$
$$\sup F_C(x) = \min\{\sup F_A(x), \sup F_B(x)\}$$

for all $x \in X$.

Definition 2.7. The intersection of two IVNSs A and B, written as $C = A \cap B$, is defined as follow

$$\inf T_C(x) = \min\{\inf T_A(x), \inf T_B(x)\}$$
$$\sup T_C(x) = \min\{\sup T_A(x), \sup T_B(x)\}$$
$$\inf I_C(x) = \max\{\inf I_A(x), \inf I_B(x)\}$$
$$\sup I_C(x) = \max\{\sup I_A(x), \sup I_B(x)\}$$
$$\inf F_C(x) = \max\{\inf F_A(x), \inf F_B(x)\}$$
$$\sup F_C(x) = \max\{\sup F_A(x), \sup F_B(x)\}$$

for all $x \in X$.

3 Similarity measure between interval valued neutrosophic sets

Similarity measure between Interval Neutrosophic sets is defined by Ye [11] as follow.

Definition 3.1. Let $N(X)$ be all IVNSs on X and $A, B \in N(X)$. A similarity measure between two IVNSs is a function $S: N(X) \times N(X) \to [0,1]$ which is satisfies the following conditions:

i. $0 \le S(A,B) \le 1$
ii. $S(A,B) = 1$ iff $A = B$
iii. $S(A,B) = S(B,A)$
iv. If $A \subset B \subset C$ then $S(A,C) \le S(A,B)$ and $S(A,C) \le S(B,C)$ for all $A, B, C \in N(X)$.

We generalize the similarity measure of intuitionistic fuzzy set, which defined in [4], to interval valued neutrosophic sets as follow.

Definition 3.2. Let A, B be two IVN sets in X. The similarity measure between the IVN sets A and B can be evaluated by the function,

$$\begin{aligned} S(A,B) = 1 - 1/2 \Big[& \max_i(|\inf T_A(x_i) - \inf T_B(x_i)|) \\ & + \max_i(|\sup T_A(x_i) - \sup T_B(x_i)|) \\ & + \max_i(|\inf I_A(x_i) - \inf I_B(x_i)|) \\ & + \max_i(|\sup I_A(x_i) - \sup I_B(x_i)|) \\ & + \max_i(|\inf F_A(x_i) - \inf F_B(x_i)|) \\ & + \max_i(|\sup F_A(x_i) - \sup F_B(x_i)|) \Big] \end{aligned}$$

for all $x \in X$.

We shall prove this similarity measure satisfies the properties of the Definition 3.1.

Proof: We show that $S(A,B)$ satisfies all properties 1-4 as above. It is obvious, the properties 1-3 is satisfied of definition 3.1. In the following we only prove 4.

Let $A \subset B \subset C$, the we have

$$\inf T_A(x_i) \le \inf T_B(x_i) \le \inf T_C(x_i)$$
$$\sup T_A(x_i) \le \sup T_B(x_i) \le \sup T_C(x_i)$$
$$\inf I_A(x_i) \ge \inf I_B(x_i) \ge \inf I_C(x_i)$$
$$\sup I_A(x_i) \ge \sup I_B(x_i) \ge \sup I_C(x_i)$$
$$\inf F_A(x_i) \ge \inf F_B(x_i) \ge \inf F_C(x_i)$$
$$\sup F_A(x_i) \ge \sup F_B(x_i) \ge \sup F_C(x_i)$$

for all $x \in X$. It follows that

$$|\inf T_A(x_i) - \inf T_B(x_i)| \le |\inf T_A(x_i) - \inf T_C(x_i)|$$
$$|\sup T_A(x_i) - \sup T_B(x_i)| \le |\sup T_A(x_i) - \sup T_C(x_i)|$$
$$|\inf I_A(x_i) - \inf I_B(x_i)| \le |\inf I_A(x_i) - \inf I_C(x_i)|$$
$$|\sup I_A(x_i) - \sup I_B(x_i)| \le |\sup I_A(x_i) - \sup I_C(x_i)|$$
$$|\inf F_A(x_i) - \inf F_B(x_i)| \le |\inf F_A(x_i) - \inf F_C(x_i)|$$
$$|\sup F_A(x_i) - \sup F_B(x_i)| \le |\sup F_A(x_i) - \sup F_C(x_i)|.$$

It means that $S(A,C) \le S(A,B)$. Similarly, it seems that $S(A,C) \le S(B,C)$.

The proof is completed

4 Entropy of an interval valued neutrosophic set

The entropy measure on IVIF sets is given by Wei [10]. We extend the entropy measure on IVIF set to interval valued neutrosophic set.

Definition 4.1. Let $N(X)$ be all IVNSs on X and $A \in N(X)$. An entropy on IVNSs is a function $E_N: N(X) \to [0,1]$ which is satisfies the following axioms:

i. $E_N(A) = 0$ if A is crisp set
ii. $E_N(A) = 1$ if $[\inf T_A(x), \sup T_A(x)] = [\inf F_A(x), \sup F_A(x)]$ and $\inf I_A(x) = \sup I_A(x)$ for all $x \in X$
iii. $E_N(A) = E_N(A^c)$ for all $A \in N(X)$
iv. $E_N(A) \ge E_N(B)$ if $A \subseteq B$ when $\sup I_A(x) - \sup I_B(x) < \inf I_A(x) - \inf I_B(x)$ for all $x \in X$.

Definition 4.2. The entropy of IVNS set A is,

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} \left[\frac{2 - |inf\, T_A(x) - inf\, F_A(x)|}{2 + |inf\, T_A(x) - inf\, F_A(x)|} \right.$$
$$\left. \frac{-|sup\, T_A(x) - sup\, F_A(x)| - |inf\, I_A(x) - sup\, I_A(x)|}{+|sup\, T_A(x) - sup\, F_A(x)| + |inf\, I_A(x) - sup\, I_A(x)|} \right]$$

for all $x \in X$.

Theorem: The IVN entropy of $E_N(A)$ is an entropy measure for IVN sets.

Proof: We show that the $E_N(A)$ satisfies the all properties given in Definition 4.1.

i. When A is a crisp set, i.e.,

$$[\inf T_A(x), \sup T_A(x)] = [0,0]$$
$$[\inf I_A(x), \sup I_A(x)] = [0,0]$$
$$[\inf F_A(x), \sup F_A(x)] = [1,1]$$

or

$$[\inf T_A(x), \sup T_A(x)] = [1,1]$$
$$[\inf I_A(x), \sup I_A(x)] = [0,0]$$
$$[\inf F_A(x), \sup F_A(x)] = [0,0]$$

for all $x_i \in X$. It is clear that $E_N(A) = 0$.

ii. Let
$[\inf T_A(x), \sup T_A(x)] = [\inf F_A(x), \sup F_A(x)]$
and $\inf I_A(x) = \sup I_A(x)$ for all $x \in X$. Then

$$E_N(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{2-0-0-0}{2+0+0+0} = \frac{1}{n}\sum_{i=1}^{n} 1 = 1.$$

iii. Since $T_{A^c}(x) = F_A(x)$, $\inf I_{A^c}(x) = 1 - \sup I_A(x)$, $\sup I_{A^c}(x) = 1 - \inf I_A(x)$ and $F_{A^c}(x) = T_A(x)$, it is clear that $E_N(A) = E_N(A^c)$.

iv. If A$\subseteq B$, then $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$ and $\sup F_A(x) \geq \sup F_B(x)$. So
$|\inf T_A(x) - \inf F_A(x)| \leq |\inf T_B(x) - \inf F_B(x)|$
and
$|\sup T_A(x) - \sup F_A(x)| \leq |\sup T_B(x) - \sup F_B(x)|$
for all $x \in X$. And
$|\inf I_A(x) - \sup I_A(x)| \leq |\inf I_B(x) - \sup I_B(x)|$
when
$\sup I_A(x) - \sup I_B(x) \leq \inf I_A(x) - \inf I_B(x)$
for all $x \in X$. Therefore it is clear that $E(A) \geq E(B)$.

The proof is completed

Conclusion

In this paper we introduced similarity measure of interval valued neutrosophic sets. These measures are consistent with similar considerations for other sets. Then we give entropy of an interval valued neutrosophic set. This entropy was generalized the entropy measure on interval valued intuitionistic sets in [10].

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Topological structures of fuzzy neutrosophic rough sets

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Abstract. In this paper, we examine the fuzzy neutrosophic relation having a special property that can be equivalently characterised by the essential properties of the lower and upper fuzzy neutrosophic rough approximation operators. Further, we prove that the set of all lower approximation sets based on fuzzy neutrosophic equivalence approximation space forms a fuzzy neutrosophic topology. Also, we discuss the necessary and sufficient conditions such that the FN interior (closure) equals FN lower (upper) approximation operator.

Keywords: Fuzzy neutrosophic rough set, Approximation operators, approximation spaces, rough sets, topological spaces.

1 Introduction

A rough set, first described by Pawlak, is a formal approximation of a crisp set in terms of a pair of sets which give the lower and the upper approximation of the original set. The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approach is based on the fuzzy set notion proposed by L. Zadeh. Rough set theory proposed by Z. Pawlak in [10] presents still another attempt to this problem. Rough sets have been proposed for a very wide variety of applications. In particular, the rough set approach seems to be important for Artificial Intelligence and cognitive sciences, especially in machine learning, knowledge discovery, data mining, expert systems, approximate reasoning and pattern recognition.

Neutrosophic Logic has been proposed by Florentine Smarandache [11, 12] which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.

In this paper we focus on the study of the relationship between fuzzy neutrosophic rough approximtion operators and fuzzy neutrosophic topological spaces.

2 Preliminaries

Definition2.1 [1]:
A fuzzy neutrosophic set A on the universe of discourse X is defined as
$A=\{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$
where $T, I, F: X \rightarrow [0, 1]$ and
$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2[1]:
A fuzzy neutrosophic relation U is a fuzzy neutrosophic subset $R=\{<x, y>, T_R(x,y), I_R(x,y), F_R(x,y) / x, y \in U\}$
$T_R: U \times U \rightarrow [0,1]$, $I_R: U \times U \rightarrow [0,1]$, $F_R: U \times U \rightarrow [0,1]$
Satisfies $0 \leq T_R(x,y) + I_R(x,y) + F_R(x,y) \leq 3$ for all $(x,y) \in U \times U$.

Definition 2.3[4]:
Let U be a non empty universe of discourse. For an arbitrary fuzzy neutrosophic relation R over $U \times U$ the pair (U, R) is called fuzzy neutrosophic approximation space. For any $A \in FN(U)$, we define the upper and lower approximation with respect to (U, R), denoted by $\overline{R}$ and $\underline{R}$ respectively.

$$\overline{R}(A) = \{< x, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x) > / x \in U\}$$

$$\underline{R}(A) = \{< x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) > x \in U\}$$

$$T_{\overline{R}(A)}(x) = \bigvee_{y \in U} [T_R(x,y) \wedge T_A(y)]$$

$$I_{\overline{R}(A)}(x) = \bigvee_{y \in U} [I_R(x,y) \wedge I_A(y)]$$

$$F_{\overline{R}(A)}(x) = \bigwedge_{y \in U} [F_R(x,y) \wedge T_A(y)]$$

$$T_{\underline{R}(A)}(x) = \bigwedge_{y\in U} [F_R(x,y) \wedge T_A(y)]$$

$$I_{\underline{R}(A)}(x) = \bigwedge_{y\in U} [1 - I_R(x,y) \wedge I_A(y)]$$

$$F_{\underline{R}(A)}(x) = \bigvee_{y\in U} [T_R(x,y) \wedge F_A(y)]$$

The pair $(\underline{R}, \overline{R})$ is fuzzy neutrosophic rough set of A with respect to (U,R) and $\overline{R}, \underline{R}$:FN(U)→FN(U) are refered to as upper and lower Fuzzy neutrosophic rough approximation operators respectively.

Theorem2.4[4]:
Let (U, R) be fuzzy neutrosophic approximation space. And $A \in FN(U)$, the upper FN approximation operator can be represented as follows $\forall\, x \in U$,

(1) $T_{\overline{R}(A)}(u) = \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R_\alpha}(A_\alpha)\,(x)]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R_\alpha}\,(A_{\alpha+})\,(x)]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R_{\alpha+}}(A_\alpha)\,(x)]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R_{\alpha+}}(A_{\alpha+})\,(x)]$

(2) $I_{\overline{R}(A)}(u) = \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R\alpha}(A\alpha)\,(x)]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R\alpha}(A\alpha_{+})\,(x)]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R\alpha}(A\alpha)\,(x)]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \wedge \overline{R\alpha+}(A\alpha_{+})\,(x)]$

(3) $F_{\overline{R}(A)}(u) = \bigwedge_{\alpha\in[0,1]} [\alpha \bigvee (1- \overline{R^\propto}(A^\alpha)\,(x))]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \bigvee (1- \overline{R^\propto}(A^{\alpha^+})(x))]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \bigvee (1- \overline{R^{\propto+}}(A^\alpha)(x))]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \bigvee (1- \overline{R^{\propto+}}(A^{\alpha^+})(x))]$

(3) $[\overline{R}(A)]_{\alpha+} \subseteq \overline{R_{\alpha+}}(A_{\alpha+}) \subseteq \overline{R_{\alpha+}}(A_\alpha) \subseteq \overline{R_{(\alpha)}}(A_\alpha) \subseteq [\overline{R}(A)]_\alpha$
(4) $[\overline{R}(A)]\alpha+ \subseteq \overline{R\propto+}(A\alpha+) \subseteq \overline{R\propto+}(A\alpha) \subseteq \overline{R\propto}(A\alpha) \subseteq [\overline{R}(A)]\alpha$
(6) $[\overline{R}(A)]^{\alpha^+} \subseteq \overline{R^{\propto+}}(A^{\alpha^+}) \subseteq \overline{R^{\propto+}}(A^\alpha) \subseteq \overline{R^\propto}(A^\alpha) \subseteq [\overline{R}(A)]_\alpha$

Theorem2.5[4]: Let (U, R) be fuzzy neutrosophic approximation space. And $A \in FN(U)$, the upper FN approximation operator can be represented as follows $\forall\, x \in U$,

(1) $T_{\underline{R}(A)}(u) = \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R^\propto}(A_{\alpha+})\,(x)]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R^\propto}(A_\alpha)\,(x)]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R^{\propto+}}(A_{\alpha+})\,(x)]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \overline{R_{\alpha+}}(A_{\alpha+})\,(x)]$

(2) $I_{\underline{R}(A)}(u) = \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R(1-\propto)}(A\alpha+)\,(x)]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R(1-\propto)}A\alpha)\,(x)]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R(1-\propto+)}(A\alpha+)\,(x)]$
$= \bigwedge_{\alpha\in[0,1]} [\alpha \vee 1\text{-}\, \underline{R(1-\propto+)}(A\alpha+)\,(x)]$

(3) $F_{\underline{R}(A)}(u) = \bigvee_{\alpha\in[0,1]} [\alpha \bigvee (1\text{-}\, \underline{R_\propto}(A^\alpha)\,(x))]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \bigvee (1\text{-}\, \underline{R_\propto}(A^{\alpha^+})(x))]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \bigvee (1\text{-}\, \underline{R_{\propto+}}(A^{\alpha^+})(x))]$
$= \bigvee_{\alpha\in[0,1]} [\alpha \bigvee (1\text{-}\, \underline{R_{\propto+}}(A^\alpha)(x))]$

(3) $[\underline{R}(A)]_{\alpha+} \subseteq \underline{R^\propto}(A_{\alpha+}) \subseteq \underline{R^{\propto+}}(A_{\alpha+}) \subseteq \underline{R^{\propto+}}(A_\alpha) \subseteq [\underline{R}(A)]_\alpha$
(4) $[\underline{R}(A)]\alpha+ \subseteq \overline{R1-\propto}(A\alpha+) \subseteq \overline{R1-\propto+}(A\alpha) \subseteq \underline{R}1-\propto+(A\alpha) \subseteq [\overline{R}(A)]\alpha$
(6) $[\underline{R}(A)]^{\alpha^+} \subseteq \underline{R_\propto}(A^{\alpha^+}) \subseteq \underline{R_{\propto+}}(A^{\alpha^+}) \subseteq \underline{R_{\propto+}}(A^\alpha) \subseteq [\underline{R}(A)]_\alpha$

3. Equivalence relation on fuzzy neutrosophic rough sets

In this section we tend to prove the fuzzy neutrosophic relation having a special property such as reflexivity and transitivity, can be equivalentely characterised by the essential properties of lower and upper approximation operators.

Theorem 3.1:
Let R be a fuzzy neutrosophic relation on U and $\overline{R}$ and $\underline{R}$ the lower and upper approximation operators induced by (U, R). Then

(1) R is reflexive ⇔
R1) $\underline{R}$ (A)⊆ A , ∀ A ∈ FN(U) ,
R2) A ⊆ $\overline{R}$ (A), ∀ A ∈ FN(U).
(2) R is symmetric ⇔

S1) $T_{\overline{R}(1_x)}(y) = T_{\overline{R}(1_y)}(x)$, ∀ (x,y)∈ U ×U,
S2) $I_{\overline{R}(1_x)}(y) = I_{\overline{R}(1_y)}(x)$, ∀ (x,y) ∈ U ×U,
S3) $F_{\overline{R}(1_x)}(y) = F_{\overline{R}(1_x)}(y)$, ∀ (x,y)∈ U ×U,
S4)$T_{\underline{R}(1_{U-\{x\}})}(y) = T_{\underline{R}(1_{U-\{y\}})}(x)$, ∀ (x,y)∈ U ×U,
S5)$I_{\underline{R}(1_{U-\{x\}})}(y) = I_{\underline{R}(1_{U-\{y\}})}(x)$, ∀ (x,y)∈ U ×U,
S6) $F_{\underline{R}(1_{U-\{x\}})}(y) = F_{\underline{R}(1_{U-\{y\}})}(x)$, ∀ (x,y)∈ U ×U.

(3) R is transitive ⇔
T1) $\underline{R}$ (A) ⊆ $\underline{R}$ ($\underline{R}$ (A)) ∀ A ∈ FN(U)
T2) $\overline{R}$ ($\overline{R}$ (A)) ⊆ $\overline{R}$ (A), ∀ A ∈ FN(U)

Proof:

(1)R1 and R2 are equivalent because of the duality of the lower and upper fuzzy neutrosophic rough approximation operators. We need to prove that reflexivity of R is equivalent to R2.

Assume that R is reflexive. For any A ∈ FN(U) and x ∈ U, by the reflexivity of R we have $T_R(x,x) = 1$, $I_R(x,x) = 1$, $F_R(x,x) = 0$. Then

$$T_{\overline{R}(A)}(x) = \bigvee_{y\in U}[T_R(x,y) \wedge T_A(y)]$$

$$\geq T_R(x,x) \wedge T_A(x) = T_A(x)$$

$$I_{\overline{R}(A)}(x) = \bigvee_{y\in U}[I_R(x,y) \wedge I_A(y)]$$

$$\geq I_R(x,x) \wedge I_A(x) = I_A(x)$$

$$F_{\overline{R}(A)}(x) = \bigwedge_{y\in U}[F_R(x,y) \vee F_A(y)]$$

$$\geq F_R(x,x) \vee F_A(x) = F_A(x)$$

Thus A ⊆ $\overline{R}$ (A), ∀ A ∈ FN(U). R2 holds.

Conversely, assume that R2 holds.

For any $x \in U$, since $A \subseteq \overline{R}(A)$ for all $A \in FN(U)$.

Let A= 1_x , we have

$$1 = T_{1_x}(x) \leq T_{\overline{R}(1_x)}(x) = \bigvee_{y\in U}[T_R(x,y) \wedge T_{1x}(y)]$$

$$= T_R(x,x)$$

$$1 = I_{1_x}(x) \leq I_{\overline{R}(1_x)}(x) = \bigvee_{y\in U}[I_R(x,y) \wedge I_{1x}(y)]$$

$$= I_R(x,x)$$

$$0 = F_{1_x}(x) \geq F_{\overline{R}(1_x)}$$

$$\bigwedge_{y\in U}[F_R(x,y) \vee F_{1_x}(y)] = F_R(x,x)$$

Hence,

$T_R(x,x) = 1$, $I_R(x,x) = 1$, $F_R(x,x) = 0$.

Thus we can conclude, that FN relation R is reflexive.

(2) For any $(x,y) \in U \times U$, we have

$$T_{\overline{R}(1_y)}(x) = \bigvee_{y'\in U}[T_R(x,y') \wedge T_{1_y}(y')] = T_R(x,y)$$

$$I_{\overline{R}(1_y)}(x) = \bigvee_{y'\in U}[I_R(x,y') \wedge I_{1_y}(y')] = I_R(x,y)$$

$$F_{\overline{R}(1_y)}(x) = \bigwedge_{y'\in U}[F_R(x,y') \vee F_{1_y}(y') = F_R(x,y)$$

Also, we have

$$T_{\overline{R}(1_x)}(y) = \bigvee_{y'\in U}[T_R(y,y') \wedge T_{1x}(y') = T_R(y,x)$$

$$I_{\overline{R}(1_x)}(y) = \bigvee_{y'\in U}[I_R(y,y') \wedge I_{1x}(y') = I_R(y,x)$$

$$F_{\overline{R}(1_x)}(y) = \bigwedge_{y'\in U}[F_R(y,y') \vee F_{1x}(y') = F_R(y,x)$$

We know, R is symmetric if and only if $T_R(x,y) = T_R(y,x)$, $I_R(x,y) = I_R(y,x)$, $F_R(x,y) = F_R(y,x)$ and S1, S2, S3 holds and similarly we can prove R is symmetric if and only if S4, S5, S6 holds.

(3) It can be easily verified that T1 and T2 are equivalent. We claim to prove that transitivity of R is equivalent to T2. Assume that R is transitive and $A \in FN(U)$. For any $x, y, z \in U$, we have

$$T_R(x,z) \geq \bigvee_{y\in U}[T_R(x,y) \wedge T_R(y,z)]$$

$$I_R(x,z) \geq \bigvee_{y\in U}[I_R(x,y) \wedge I_R(y,z)]$$

$$F_R(x,z) \leq \bigvee_{y\in U}[F_R(x,y) \wedge F_R(y,z)] .$$

We obtain,

$$T_{\overline{R}(\overline{R}(A))}(x) = \bigvee_{y\in U}[T_R(x,y) \wedge T_{\overline{R}(A)}(y)]$$

$$= \bigvee_{y\in U}[T_R(x,y) \wedge \bigvee_{z\in U}[T_R(y,z) \wedge T_A(z)]$$

$$= \bigvee_{y\in U}\bigvee_{z\in U}[T_R(x,y) \wedge T_R(y,z) \wedge T_A(z)]$$

$$= \bigvee_{y\in U}[\bigvee_{z\in U}(T_R(x,y) \wedge T_R(y,z)) \wedge T_A(z)]$$

$$\leq \bigvee_{z\in U}[T_R(x,z) \wedge T_A(z) = T_{\overline{R}(A)}(x)$$

$$I_{\overline{R}(\overline{R}(A))}(x) = \bigvee_{y\in U}[I_R(x,y) \wedge I_{\overline{R}(A)}(y)]$$

$$= \bigvee_{y\in U}[I_R(x,y) \wedge \bigvee_{z\in U}[I_R(y,z) \wedge I_A(z)]$$

$$= \bigvee_{y\in U}\bigvee_{z\in U}[I_R(x,y) \wedge I_R(y,z) \wedge I_A(z)]$$

$$= \underset{y\in U}{\vee}[\underset{z\in U}{\vee}(I_R(x,y)\wedge I_R(y,z))\wedge I_A(z)]$$
$$\leq \underset{z\in U}{\vee}[I_R(x,z)\wedge I_A(z) = I_{\overline{R}(A)}(x)$$

$$F_{\overline{R}(\overline{R}(A))}(x) = \underset{y\in U}{\wedge}[F_R(x,y)\vee F_{\overline{R}(A)}(y)]$$
$$= \underset{y\in U}{\wedge}[F_R(x,y)\vee \underset{z\in U}{\wedge}[F_R(y,z)\vee F_A(z)]$$
$$= \underset{y\in U}{\wedge}\underset{z\in U}{\wedge}[F_R(x,y)\vee F_R(y,z)\vee F_A(z)]$$
$$= \underset{y\in U}{\wedge}[\underset{z\in U}{\wedge}(F_R(x,y)\vee F_R(y,z))\vee F_A(z)]$$
$$\geq \underset{z\in U}{\wedge}[F_R(x,z)\vee F_A(z) = F_{\overline{R}(A)}(x)$$

Thus, $\overline{R}$ ($\overline{R}$ (A)) ⊆ $\overline{R}$ (A), ∀ A ∈ FN(U), T2 holds.

Conversely, assume that T2 holds, For any $x, y, z \in U$

And $\lambda_1, \lambda_2, \lambda_3 \in [0,1]$, if $T_R(x,y) \geq \lambda_1$, $T_R(y,z) \geq \lambda_1$ $I_R(y,z) \geq \lambda_2$ $I_R(y,z) \geq \lambda_2$, $F_R(x,y) \leq \lambda_3$, $F_R(x,y) \leq \lambda_3$ then by T2, we have

$$T_{\overline{R}(\overline{R}(1_z))}(x) \leq T_{\overline{R}(1_z)}(x)$$
$$= \underset{y\in U}{\vee}[T_R(x,y)\wedge T_{1_z}(y)] = T_R(x,z).$$
$$I_{\overline{R}(\overline{R}(1_z))}(x) \leq I_{\overline{R}(1_z)}(x)$$
$$= \underset{y\in U}{\vee}[I_R(x,y)\wedge I_{1_z}(y)] = I_R(x,z).$$
$$F_{\overline{R}(\overline{R}(1_z))}(x) \geq F_{\overline{R}(1_z)}(x)$$
$$= \underset{y\in U}{\wedge}[F_R(x,y)\vee F_{1_z}(y)] = F_R(x,z).$$

On otherhand,

$$T_{\overline{R}(\overline{R}(1_z))}(x) = \underset{\alpha\in[0,1]}{\vee}[\alpha\wedge\overline{R}_\alpha(\overline{R}(1_z))_\alpha(x)]$$
$$= \sup\{\alpha\in[0,1]\bigcap x\in\overline{R}_\alpha(\overline{R}(1_z))_\alpha\}$$
$$= \sup\{\alpha\in[0,1]\bigcap\overline{R}_\alpha\cap(\overline{R}(1_z))_\alpha\neq\phi\}$$
$$= \sup\{\alpha\in[0,1]/\exists u\in U[T_R(x,u)\geq\alpha, T_{\overline{R}(1_z)}(\alpha)\geq\alpha]\}$$
$$= \sup\{\alpha\in[0,1]/\exists u\in U[T_R(x,u)\geq\alpha, T_R(u,z)\geq\alpha]\}$$
$$\geq T_R(x,y)\wedge T_R(y,z)\geq\lambda_1$$

Thus we obtain $T_R(x,z)\geq\lambda_1$,and

$$I_{\overline{R}(\overline{R}(1_z))}(x) = \underset{\alpha\in[0,1]}{\vee}[\alpha\wedge\overline{R}_\alpha(\overline{R}(1_z))_\alpha(x)]$$
$$= \sup\{\alpha\in[0,1]\bigcap x\in\overline{R}_\alpha(\overline{R}(1_z))_\alpha\}$$
$$= \sup\{\alpha\in[0,1]\bigcap\overline{R}_\alpha\cap(\overline{R}(1_z))_\alpha\neq\phi\}$$
$$= \sup\{\alpha\in[0,1]/\exists u\in U[I_R(x,u)\geq\alpha, I_{\overline{R}(1_z)}(\alpha)\geq\alpha]\}$$
$$= \sup\{\alpha\in[0,1]/\exists u\in U[I_R(x,u)\geq\alpha, I_R(u,z)\geq\alpha]\}$$
$$\geq I_R(x,y)\wedge I_R(y,z)\geq\lambda_2$$

Thus we obtain $I_R(x,z)\geq\lambda_2$. also

$$F_{\overline{R}(\overline{R}(1_z))}(x) = \underset{\alpha\in[0,1]}{\wedge}[\alpha\vee\overline{R}_\alpha(\overline{R}(1_z))_\alpha(x)]$$
$$= \inf\{\alpha\in[0,1]\bigcap x\in\overline{R}_\alpha(\overline{R}(1_z))_\alpha\}$$
$$= \inf\{\alpha\in[0,1]\bigcap\overline{R}_\alpha\cap(\overline{R}(1_z))_\alpha\neq\phi\}$$
$$= \inf\{\alpha\in[0,1]/\exists u\in U[F_R(x,u)\geq\alpha, F_{\overline{R}(1_z)}(\alpha)\geq\alpha]\}$$
$$= \inf\{\alpha\in[0,1]/\exists u\in U[F_R(x,u)\geq\alpha, F_R(u,z)\geq\alpha]\}$$
$$\geq F_R(x,y)\wedge F_R(y,z)\geq\lambda_3$$

Thus $F_R(y,z)\leq\lambda_3$.

Hence, FN relation is transitive.

Corollary 3.2:
Let (U,R) be a fuzzy neutrosophic reflexive and transitive aproxiation space, i.e R is a fuzzy neutrosophic reflexive and transitive relation on U, and $\underline{R}$ and $\overline{R}$ the lower and upper FN rough approximation operator induced by (U,R). Then

(RT1) $\underline{R}(A) = \underline{R}(\underline{R}(A))\forall FN(U)$

(RT2) $\overline{R}(\overline{R}(A)) = \overline{R}(A)$

4. Relation between fuzzy neutrosophic approximation spaces and fuzzy neutrosophic topological spaces.

In this section, we generalise Fuzzy neutrosophic rough set theory in fuzzy neutrosophic topological spaces and investigate the relations between fuzzy neutrosophic rough set approximation and topologies.

4.1. From a fuzzy neutrosophic approximation space to fuzzy neutrosophic topological space

In this subsection, we assume that $U\neq\phi$ is a universe of discourse, R a fuzzy neutrosophic reflexive and transitive binary relation on U and $\underline{R}$ and $\overline{R}$ the lower and upper FN rough approximation operator induced by (U,R).

Theorem 4.1.1:

Let J be an index set, $A_j \in FN(U)$. Then

$$\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) = \underset{j\in J}{\cup}\underline{R}(A_j).$$

Proof:
By reflexivity of R and Theorem (3.1), we have
$\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) \subseteq \underset{j\in J}{\cup}\underline{R}(A_j)$.
Since $\underset{j\in J}{\cup}\underline{R}(A_j) \supseteq \underline{R}(A_j)$, for all $j \in J$.
We have, $\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) \supseteq \underline{R}(\underline{R}(A_j))$
By transitivity of Rand theorem(3.1)
$\underline{R}(\underline{R}(A_j)) \supseteq \underline{R}(A_j)$.
Thus $\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) \subseteq \underline{R}(A_j)$, for all $j \in J$.
Consequently,
$\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) \supseteq \underset{j\in J}{\cup}\underline{R}(A_j)$
Hence we conclude
$\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) = \underset{j\in J}{\cup}\underline{R}(A_j)$.

Theorem 4.1.2:
$\tau_R = \{\underline{R}(A) / A \in FN(U)\}$ is a fuzzy neutrosophic toplogy on U.
Proof:
(I) In terms of Theorem (1) [4] we have $\underline{R}(1\sim) = 1\sim$, thus $1\sim \in \tau_R$ Since R is reflexive, by theorem 3.1, we have
$\underline{R}(0\sim) = 0\sim$,therefore $0\sim \in \tau_R$,
(II) $\forall$ A, B $\in$ FN(U), since $\underline{R}(A), \underline{R}(B) \in \tau_R$ by theorem (1) [4] we have $\underline{R}(A) \cap \underline{R}(B) = \underline{R}(A \cap B) \in \tau_R$
(III) $\forall A_j \in FN(U), j \in J, J$ is an index set, by theorem 4.1.1 we have
$\underline{R}(\underset{j\in J}{\cup}\underline{R}(A_j)) = \underset{j\in J}{\cup}\underline{R}(A_j)$.
Thus $\underset{j\in J}{\cup}\underline{R}(A_j) \in \tau_R$.
Therefore, $\tau_R = \{\underline{R}(A) / A \in FN(U)\}$ is a fuzzy neutrosophic toplogy on U.

Therefore Theorem 4.1.2 states that a fuzzy neutrosophic reflexive and transitive approximation space can generate fuzzy neutrosophic topolgical space such that the family of all lower approximations of fuzzy neutrosophic sets with respect to fuzzy neutrosophic approximation space forms fuzzy neutrosophic topology.

Theorem 4.1.3:
Let (U, τ_R) be the fuzzy neutrosophic topological space induced from a fuzzy neutrsophic reflexive and transitive approximation space (U,R), i.e
$\tau_R = \{\underline{R}(A) / A \in FN(U)\}$. Then, $\forall A \in FN(U)$.

$1)\underline{R}(A) = \text{int}(A) = \cup\{\underline{R}(B) \cap R(B) \subseteq A, B \in FN(U)\}$
$2)\overline{R}(A) = cl(A) = \cap\{\sim \underline{R}(B) \cap \sim \underline{R}(B) \supseteq A, B \in FN(U)\}$
$= \cap\{\overline{R}(B) \cap \overline{R}(B) \supseteq A, B \in FN(U)\}$

Proof:
(1) Since R is reflexive, by Theorem 3.1, we have
$\underline{R}(A) \subseteq A$.
Thus $\underline{R}(A) \subseteq \cup\{\underline{R}(B) \cap R(B) \subseteq A, B \in FN(U)\}$.
On other hand $\cup\{\underline{R}(B) \cap R(B)\} \subseteq A$,
then by Theorem 3.1
We obtain $\underline{R}(\cup\{\underline{R}(B) \cap R(B)\}) \subseteq \underline{R}(A)$ .In terms of Theorem 3.2 we concude $\cup\{\underline{R}(B) \cap R(B) \subseteq A\}$
$\cup\{\underline{R}(B) \cap R(B) \subseteq A\} = \underline{R}(\cup\{\underline{R}(B) \cap R(B) \subseteq A\})$
Hence, $\underline{R}(A) = \text{int}(A) = \cup\{\underline{R}(B) \cap R(B) \subseteq A\}$

(2) Follows from the duality of $\overline{R}$ and $\underline{R}$ and (1)

Theorem 4.1.4 :
Let (U, R) be a fuzzy neutrosophic reflexive and transitive approxiatin space and (U, τ) the fuzzy neutrosophic topological space induced by (U,R). Then
$T_R(x,y) = \bigwedge_{B\in y_\tau} T_B(x)$, $I_R(x,y) = \bigwedge_{B\in y_\tau} I_B(x)$,

$F_R(x,y) = \bigvee_{B\in y_\tau} F_B(x)$, $\forall x, y \in U$.

Where
$(y)_\tau = \{B \in FN(U) \cap \sim B \in \tau_R,$
$T_B(y) = 1, I_B(y) = 1, F_B(y) = 0\}$
Proof:
For any $x, y \in U$, by Thm 4.1.2 we have
$\overline{R}(1_y) = cl(1_y)$.
Also, $T_{\overline{R}(1_y)}(x) = \underset{u\in U}{\vee}[T_R(x,u) \wedge T_{1_y}(u)] = T_R(x,y)$
$I_{\overline{R}(1_y)}(x) = \underset{u\in U}{\vee}[I_R(x,u) \wedge I(u)] = I_R(x,y)$
$F_{\overline{R}(1_y)}(x) = \underset{u\in U}{\wedge}[F_R(x,u) \vee F_{1_y}(u)] = F_R(x,y)$
On other hand $cl(1_y)$
$= \cap\{B \in FN(U) \cap B\}$ is a FN closed set and $1_y \subseteq B\}$
$= \cap\{B \in FN(U) \cap \sim B \in \tau_R\}$ and $1_y \subseteq B\}$
Then

$$\mathrm{T}_{cl(1_y)}(x) = \wedge\{T_B(x) \bigcap \sim B \in \tau_R, B \supseteq 1_y\}$$
$$= \wedge\{T_B(x) \bigcap \sim B \in \tau_R,$$
$$T_B(y) = 1, I_B(y) = 1, \mathrm{F}_B(y) = 0$$

$$\underset{B\in(y)_\tau}{\wedge} T_B(x)$$

$$\mathrm{I}_{cl(1_y)}(x) = \wedge\{T_B(x) \bigcap \sim B \in \tau_R, B \supseteq 1_y\}$$
$$= \wedge\{I_B(x) \bigcap \sim B \in \tau_R,$$
$$T_B(y) = 1, I_B(y) = 1, F_B(y) = 0$$

$$\underset{B\in(y)_\tau}{\wedge} I_B(x)$$

$$\mathrm{F}_{cl(1_y)}(x) = \vee\{F_B(x) \bigcap \sim B \in \tau_R, B \supseteq 1_y\}$$
$$= \vee\{F_B(x) \bigcap \sim B \in \tau_R,$$
$$T_B(y) = 1, I_B(y) = 1, \mathrm{F}_B(y) = 0$$

$$= \underset{B\in(y)_\tau}{\vee} F_B(x)$$

Hence,

$$T_R(x,y) = \underset{B\in y_\tau}{\wedge} \mathrm{T}_\mathrm{B}(x),\ I_R(x,y) = \underset{B\in y_\tau}{\wedge} I_B(x),$$

$$F_R(x,y) = \underset{B\in y_\tau}{\vee} F_B(x),\ \forall x, y \in U.$$

4.2. Fuzzy neutrsophic approximation space

In this section we discuss the suffiecient and necessary conditions under which a FN topological space be associated with a fuzzy neutrosophic approximation space and proved $cl(A) = \overline{R}(A)$ and $\mathrm{int}(A) = \underline{R}(A)$.

Definition 4.2.1:

If P:FN(U)→FN(U) is an operator from FN(U) to FN(U), we can define three operators from F(U) to F(U), denoted by P_T, P_I, P_F, such that $P_T(T_A) = T_{P(\mathrm{A})}$ and $P_T(T_A) = I_{P(\mathrm{A})}$ and $P_T(T_A) = F_{P(\mathrm{A})}$.

That is $P(A) = P((T_A, I_A, F_A))$

$= (T_{P(A)}, I_{P(A)}, F_{P(A)})$

$= P_T(T_A), P_I(I_A), P_F(F_A)$

Theorem 4.2.2:

Let (U, τ) be fuzzy neutrosophic topological space and Cl, int:FN(U)→FN(U) the fuzzy neurtrosophic closure operator and fuzzy neutrosophic interior operator respectively. Then there exits a fuzzy neutrosophic reflexive and transitive relation R on U such that $\overline{R}(A) = cl(A)$ and $\underline{R}(A) = \mathrm{int}(A)$ for all $A \in FN(U)$

Iff cl satisfies the fowing conditions (C1) and (C2), or equivalently, int satisfies the following conditions (I1) and (I2).

(I1) $cl(A \cap \overline{\overline{(\alpha,\beta,\gamma)}}) = cl(A) \cap \overline{\overline{(\alpha,\beta,\gamma)}}$

$\forall A \in FN(U), \forall \alpha, \beta, \gamma \in [0,1]$

With $\alpha + \beta + \gamma \le 3$

(I2) $\mathrm{cl}(\underset{i\in J}{\cup} A_i) = \underset{i\in J}{\cup} \mathrm{cl}(\mathrm{c}),\ A_i \in FN(U), i \in J$, J is any index set.

(C1) $\mathrm{int}(A \cup \overline{\overline{(\alpha,\beta,\gamma)}}) = \mathrm{int}(A) \cup \overline{\overline{(\alpha,\beta,\gamma)}}$

$\forall A \in FN(U), \forall \alpha, \beta, \gamma \in [0,1]$

(C2) $\mathrm{int}(\underset{i\in J}{\cap} A_i) = \underset{i\in J}{\cap} \mathrm{int}(A_i),\ A_i \in FN(U), i \in J$, J is any index set.

Proof:

Assume that there exists a fuzzy neutrosophic reflexive and transitive relation R on U such that $\overline{R}(A) = cl(A)$ and $\underline{R}(A) = \mathrm{int}(A)$ for all $A \in FN(U)$, then by theorem 3.1, it can be easily seen that (C1), (C2), (I1), (I2) easily hold.

Converesly, Assume that closure operator cl:FN(U)→FN(U) satisfies conitions (C1) and (C2) and the interior operator int:FN(U)→FN(U) satifies the conditions (I1) and (I2).

For the closure operator we derive operators cl_T, cl_I and cl_F from FN(U) to FN(U) such that $cl_T(T_A) = T_{cl(A)}$, $cl_T(I_A) = I_{cl(A)}$, $cl_T(F_A) = F_{cl(A)}$. Likewise, from the interior operator int we have three operators $\mathrm{int}_T, \mathrm{int}_I, \mathrm{int}_F$ from FN(U) to FN(U) such that $\mathrm{int}_T(T_A) = T_{\mathrm{int(A)}}$

$\mathrm{int}_T(T_A) = I_{\mathrm{int(A)}}$, $\mathrm{int}_T(T_A) = F_{\mathrm{int(A)}}$. We now define a FN relation R on U by cl as follows: for $(x,y) \in U \times U$.

$T_R(x,y) = cl_T(\mathrm{T}_{1_y})(x)$, $I_R(x,y) = cl_I(I_{1_y})(x)$, $F_R(x,y) = cl_F(F_{1_y})(x)$

For $A \in FN(U)$

$$T_A = \underset{y\in U}{\cup}[T_{1_y} \cap \overline{\overline{T_A(\mathrm{y})}}],$$

$$I_A = \underset{y\in U}{\cup}[I_{1_y} \cap \overline{\overline{I_A(\mathrm{y})}}],$$

$F_A = \underset{y\in U}{\cup}[F_{1_y} \cap \overline{\overline{F_A(y)}}]$,

We also observe that (C1) implies (CT1), (CI1) and (CF1), and (C2) implies (CT2), (CI2) and (CF2)

(CT1) $(cl_T(\mathrm{T}_{A\cap\overline{\overline{(\alpha,\beta,\gamma)}}}) = cl_T(\mathrm{T}_{A\cap\overline{\overline{(\alpha)}}}) = cl_T(T_A)\cap\overline{\overline{\alpha}}$

$\forall A \in FN(U), \forall \alpha,\beta,\gamma \in [0,1]$ with $\alpha+\beta+\gamma \le 3$.

(CI1) $(cl_I(I_{A\cap\overline{\overline{(\alpha,\beta,\gamma)}}}) = cl_I(I_{A\cap\overline{\overline{(\beta)}}}) = cl_I(I_A)\cap\overline{\overline{\beta}}$

$\forall A \in FN(U), \forall \alpha,\beta,\gamma \in [0,1]$ with $\alpha+\beta+\gamma \le 3$.

(CF1) $(cl_F(F_{A\cap\overline{\overline{(\alpha,\beta,\gamma)}}}) = cl_F(F_{A\cap\overline{\overline{(\gamma)}}}) = cl_F(F_A)\cap\overline{\overline{\gamma}}$

$\forall A \in FN(U), \forall \alpha,\beta,\gamma \in [0,1]$ with $\alpha+\beta+\gamma \le 3$.

(CT2) $cl_T(\mathrm{T}\underset{i\in J}{\cup}\mathrm{A_i}) = cl_T(\underset{i\in J}{\cup}T_{A_i})$

$= \underset{i\in J}{\cup} cl_T(T_{A_i})$, $A_i \in FN(U)$, $i \in J$, J is any index set.

(CI2) $cl_I(I\underset{i\in J}{\cup}\mathrm{A_i}) = cl_I(\underset{i\in J}{\cup}I_{A_i})$

$= \underset{i\in J}{\cup} cl_I(I_{A_i})$, $A_i \in FN(U)$, $i \in J$, J is any index set.

, J is any index set.

(CF2) $cl_F(F\underset{i\in J}{\cup}\mathrm{A_i}) = cl_F(\underset{i\in J}{\cup}F_{A_i})$

$= \underset{i\in J}{\cup} cl_F(F_{A_i})$, $A_i \in FN(U)$, $i \in J$, J is any index set.

Then for any , $x \in U$ according to definition 4.2.1, and above properties, we have

$T_{\overline{R}(\mathrm{A})}(\mathrm{x}) = \underset{y\in U}{\vee}[T_R(x,y)\wedge T_A(y))]$

$= \underset{y\in U}{\vee}[cl_T(T_{1_y})(y)\wedge T_A(y))]$

$= \underset{y\in U}{\vee}[(cl_T(T_{1_y})\cap\overline{\overline{T_A(y))}}]$

$= \underset{y\in U}{\vee}[(cl_T(T_{1_y}\cap\overline{\overline{T_A(y)}}))]$

$= [\underset{y\in U}{\cup}(cl_T(T_{1_y}\cap\overline{\overline{T_A(y)}}))](x)$

$= [(cl_T(\underset{y\in U}{\cup}(T_{1_y}\cap\overline{\overline{T_A(y)}})))](x)$

$= (cl_T(T_A)(x) = T_{cl(A)}(x)$

$I_{\overline{R}(\mathrm{A})}(\mathrm{x}) = \underset{y\in U}{\vee}[I_R(x,y)\wedge I_A(y))]$

$= \underset{y\in U}{\vee}[cl_I(I_{1_y})(y)\wedge I_A(y))]$

$= \underset{y\in U}{\vee}[(cl_I(I_{1_y})\cap\overline{\overline{I_A(y))}}]$

$= \underset{y\in U}{\vee}[(cl_I(I_{1_y}\cap\overline{\overline{I_A(y)}}))]$

$= [\underset{y\in U}{\cup}(cl_I(I_{1_y}\cap\overline{\overline{I_A(y)}}))](x)$

$= [(cl_I(\underset{y\in U}{\cup}(I_{1_y}\cap\overline{\overline{I_A(y)}})))](x)$

$= (cl_I(I_A)(x) = I_{cl(A)}(x)$

$F_{\overline{R}(\mathrm{A})}(\mathrm{x}) = \underset{y\in U}{\wedge}[F_R(x,y)\vee F_A(y))]$

$= \underset{y\in U}{\wedge}[cl_F(F_{1_y})(y)\vee F_A(y))]$

$= \underset{y\in U}{\wedge}[(cl_F(F_{1_y})\cup\overline{\overline{FI_A(y))}}]$

$= \underset{y\in U}{\wedge}[(cl_F(F_{1_y}\cup\overline{\overline{F_A(y)}}))]$

$= [\underset{y\in U}{\cap}(cl_F(F_{1_y}\cup\overline{\overline{F_A(y)}}))](x)$

$= [(cl_F(\underset{y\in U}{\cap}(F_{1_y}\cup\overline{\overline{F_A(y)}})))](x)$

$= (cl_F(F_A)(x) = F_{cl(A)}(x)$

Thus $cl(A) = \overline{R}(A)$.

Similary we can prove $\mathrm{int}(A) = \underline{R}(A)$

Conclusion:

In this paper we defined the topological structures of fuzzy neutrosophic rough sets. We found that fuzzy neutrosophic topological space can be induced by fuzzy rough approximation operator if and only if fuzzy neutrosophic relation is reflexive and transitive. Also we have investigated the sufficient and necessary condition for which a fuzzy neutrosophic topological space can associate with fuzzy neutrosophic reflexive and transitive rough approximation space such that FN rough upper approximation equals closure and FN rough lower approximation equals interior operator.

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Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies

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Abstract. In this paper, we make a short history about: the neutrosophic set, neutrosophic numerical components and neutrosophic literal components, neutrosophic numbers, neutrosophic intervals, neutrosophic hypercomplex numbers of dimension n, and elementary neutrosophic algebraic structures. Afterwards, their generalizations to refined neutrosophic set, respectively refined neutrosophic numerical and literal components, then refined neutrosophic numbers and refined neutrosophic algebraic structures. The aim of this paper is to construct examples of splitting the literal indeterminacy ($\boldsymbol{I}$) into literal sub-indeterminacies ($\boldsymbol{I_1, I_2, \dots, I_r}$), and to define a multiplication law of these literal sub-indeterminacies in order to be able to build refined $\boldsymbol{I}$ – neutrosophic algebraic structures. Also, examples of splitting the numerical indeterminacy ($\boldsymbol{i}$) into numerical sub-indeterminacies, and examples of splitting neutrosophic numerical components into neutrosophic numerical sub-components are given.

Keywords: neutrosophic set, elementary neutrosophic algebraic structures, neutrosophic numerical components, neutrosophic literal components, neutrosophic numbers, refined neutrosophic set, refined elementary neutrosophic algebraic structures, refined neutrosophic numerical components, refined neutrosophic literal components, refined neutrosophic numbers, literal indeterminacy, literal sub-indeterminacies, $\boldsymbol{I}$-neutrosophic algebraic structures.

1 Introduction

Neutrosophic Set was introduced in 1995 by Florentin Smarandache, who coined the words "neutrosophy" and its derivative „neutrosophic". The first published work on neutrosophics was in 1998 see [3].

There exist two types of neutrosophic components: numerical and literal.

2 Neutrosophic Numerical Components

Of course, the *neutrosophic numerical components* (t, i, f) are crisp numbers, intervals, or in general subsets of the unitary standard or nonstandard unit interval.

Let $\mathcal{U}$ be a universe of discourse, and M a set included in $\mathcal{U}$. A generic element x from $\mathcal{U}$ belongs to the set M in the following way: $x(t, i, f) \in M$, meaning that x's degree of membership/truth with respect to the set M is t, x's degree of indeterminacy with respect to the set M is i, and x's degree of non-membership/falsehood with respect to the set M is f, where t, i, f are independent standard subsets of the interval $[0, 1]$, or non-standard subsets of the non-standard interval $]^{-}0, 1^{+}[$ in the case when one needs to make distinctions between *absolute and relative* truth, indeterminacy, or falsehood.

Many papers and books have been published for the cases when t, i, f were single values (crisp numbers), or t, i, f were intervals.

3 Neutrosophic Literal Components

In 2003, W. B. Vasantha Kandasamy and Florentin Smarandache [4] introduced the *literal indeterminacy* "I", such that $I^2 = I$ (whence $I^n = I$ for $n \geq 1$, n integer). They extended this to *neutrosophic numbers* of the form: $a + bI$, where a, b are real or complex numbers, and

$$(a_1 + b_1 I) + (a_2 + b_2 I) = (a_1 + a_2) + (b_1 + b_2) I \quad (1)$$

$$(a_1 + b_1 I)(a_2 + b_2 I) = (a_1 a_2) + (a_1 b_2 + a_2 b_1 + b_1 b_2) I \quad (2)$$

and developed many I-neutrosophic algebraic structures based on sets formed of neutrosophic numbers.

Working with imprecisions, Vasantha Kandasamy & Smarandache have proposed (approximated) I^2 *by* I; yet different approaches may be investigated by the interested researchers where $I^2 \neq I$ (in accordance with their believe and with the practice), and thus a new field would arise in the neutrosophic theory.

The neutrosophic number $N = a + bI$ can be interpreted as: "a" represents the determinate part of number N, while "bI" the indeterminate part of number N.

For example, $\sqrt{7} = 2.6457...$ that is irrational has infinitely many decimals. We cannot work with this exact number in our real life, we need to approximate it. Hence, we

may write it as *2 + I* with *I ∈ (0.6, 0.7)*, or as *2.6 + 3I* with *I ∈ (0.01, 0.02)*, or *2.64 + 2I* with *I ∈ (0.002, 0.004)*, etc. depending on the problem to be solved and on the needed accuracy.

Jun Ye [9] applied the neutrosophic numbers to decision making in *2014*.

4 Neutrosophic Intervals

We now for the first time extend the neutrosophic number to (open, closed, or half-open half-closed) neutrosophic interval. A *neutrosophic interval A* is an (open, closed, or half-open half-closed) interval that has some indeterminacy in one of its extremes, i.e. it has the form $A = [a, b] \cup \{cI\}$, or $A = \{cI\} \cup [a, b]$, where *[a, b]* is the determinate part of the neutrosophic interval A, and *I* is the indeterminate part of it (while *a*, *b*, *c* are real numbers, and $\cup$ means union). (Herein *I* is an interval.)

We may even have neutrosophic intervals with double indeterminacy (or refined indeterminacy): one to the left (I_1), and one to the right (I_2):

$$A = \{c_1 I_1\} \cup [a, b] \cup \{c_2 I_2\}. \quad (3)$$

A classical real interval that has a neutrosophic number as one of its extremes becomes a neutrosophic interval. For example: *[0, $\sqrt{7}$]* can be represented as *[0, 2]* $\cup$ *I* with *I = (2.0, 2.7)*, or *[0, 2]* $\cup$ *{10I}* with *I = (0.20, 0.27)*, or *[0, 2.6]* $\cup$ *{10I}* with *I = (0.26, 0.27)*, or *[0, 2.64]* $\cup$ *{10I}* with *I =* (0.264, 0.265), etc. in the same way depending on the problem to be solved and on the needed accuracy.

We gave examples of closed neutrosophic intervals, but the open and half-open half-closed neutrosophic intervals are similar.

5 Notations

In order to make distinctions between the numerical and literal neutrosophic components, we start denoting the *numerical indeterminacy* by lower case letter "*i*" (whence consequently similar notations for *numerical truth* "*t*", and for *numerical falsehood* "*f*"), and *literal indeterminacy* by upper case letter "*I*" (whence consequently similar notations for *literal truth* "*T*", and for *literal falsehood* "*F*").

6 Refined Neutrosophic Components

In *2013*, F. Smarandache [3] introduced the refined neutrosophic components in the following way: the neutrosophic numerical components t, i, f can be refined (split) into respectively the following refined neutrosophic numerical sub-components:

$$\langle t_1, t_2, \dots t_p;\ i_1, i_2, \dots\ i_r;\ f_1, f_2, \dots f_s \rangle, \quad (4)$$

where p, r, s are integers ≥ 1 and $\max\{p, r, s\} \geq 2$, meaning that at least one of p, r, s is ≥ 2; and t_j represents types of numeral truths, i_k represents types of numeral indeterminacies, and f_l represents types of numeral falsehoods, for $j = 1, 2, \dots, p$; $k = 1, 2, \dots, r$; $l = 1, 2, \dots, s$.

t_j, i_k, f_l are called numerical subcomponents, or respectively *numerical sub-truths*, *numerical sub-indeterminacies*, and *numerical sub-falsehoods.*

Similarly, the neutrosophic literal components T, I, F can be refined (split) into respectively the following neutrosophic literal subcomponents:

$$\langle T_1, T_2, \dots T_p;\ I_1, I_2, \dots I_r;\ F_1, F_2, \dots F_s \rangle, \quad (5)$$

where p, r, s are integers ≥ 1 too, and $\max\{p, r, s\} \geq 2$, meaning that at least one of p, r, s is ≥ 2; and similarly T_j represent types of literal truths, I_k represent types of literal indeterminacies, and F_l represent types of literal falsehoods, for $j = 1, 2, \dots, p$; $k = 1, 2, \dots, r$; $l = 1, 2, \dots, s$.

T_j, I_k, F_l are called literal subcomponents, or respectively *literal sub-truths*, *literal sub-indeterminacies*, and *literal sub-falsehoods*.

Let consider a *simple example of refined numerical components*.

Suppose that a country C is composed of two districts D_1 and D_2, and a candidate John Doe competes for the position of president of this country C. Per whole country, $NL(\text{Joe Doe}) = (0.6, 0.1, 0.3)$, meaning that 60% of people voted for him, 10% of people were indeterminate or neutral – i.e. didn't vote, or gave a black vote, or a blank vote –, and 30% of people voted against him, where NL means the neutrosophic logic values.

But a political analyst does some research to find out what happened to each district separately. So, he does a refinement and he gets:

$$\begin{pmatrix} 0.40 & 0.20 ; & 0.08 & 0.02 ; & 0.05 & 0.25 \\ t_1 & t_2 & i_1 & i_2 & f_1 & f_2 \end{pmatrix} \quad (6)$$

which means that 40% of people that voted for Joe Doe were from district D_1, and 20% of people that voted for Joe Doe were from district D_2; similarly, 8% from D_1 and 2% from D_2 were indeterminate (neutral), and 5% from D_1 and 25% from D_2 were against Joe Doe.

It is possible, in the same example, to refine (split) it in a different way, considering another criterion, namely: what percentage of people did not vote (i_1), what percentage of people gave a blank vote – cutting all candidates on the ballot – (i_2), and what percentage of people gave a blank vote – not selecting any candidate on the ballot (i_3). Thus, the numerical indeterminacy (i) is refined into i_1, i_2, and i_3:

$$\begin{pmatrix} 0.60 ; & 0.05 & 0.04 & 0.01 ; & 0.30 \\ t & i_1 & i_2 & i_3 & f \end{pmatrix} \quad (7)$$

7 Refined Neutrosophic Numbers

In 2015, F. Smarandache [6] introduced the *refined literal indeterminacy* (I), which was split (refined) as $I_1, I_2, \dots, I_r$, with $r \geq 2$, where I_k, for $k = 1, 2, \dots, r$ represent types of literal sub-indeterminacies. A refined neutrosophic number has the general form:

$$N_r = a + b_1 I_1 + b_2 I_2 + \cdots + b_r I_r, \quad (8)$$

where $a, b_1, b_2, \dots, b_r$ are real numbers, and in this case N_r is called a *refined neutrosophic real number*; and if at least one of $a, b_1, b_2, \dots, b_r$ is a complex number (i.e. of the form $\alpha + \beta\sqrt{-1}$, with $\beta \neq 0$, and α, β real numbers), then N_r is called a *refined neutrosophic complex number*.

An example of refined neutrosophic number, with three types of indeterminacies resulted from the cubic root (I_1), from Euler's constant e (I_2), and from number π (I_3):

$$N_3 = -6 + \sqrt[3]{59} - 2e + 11\pi \quad (9)$$

Roughly

$N_3 = -6 + (3 + I_1) - 2(2 + I_2) + 11(3 + I_3)$

$= (-6 + 3 - 4 + 33) + I_1 - 2I_2 + 11I_3 = 26 + I_1 - 2I_2 + 11I_3$

where $I_1 \in (0.8, 0.9)$, $I_2 \in (0.7, 0.8)$, and $I_3 \in (0.1, 0.2)$, since $\sqrt[3]{59} = 3.8929\dots$, $e = 2.7182\dots$, $\pi = 3.1415\dots$.

Of course, other 3-valued refined neutrosophic number representations of N_3 could be done depending on accuracy.

Then F. Smarandache [6] defined the *refined I-neutrosophic algebraic structures* in 2015 as algebraic structures based on sets of refined neutrosophic numbers.

Soon after this definition, Dr. Adesina Agboola wrote a paper on refined *I*-neutrosophic algebraic structures [7].

They were called "*I*-neutrosophic" because the refinement is done with respect to the literal indeterminacy (I), in order to distinguish them from the refined (t, i, f)-neutrosophic algebraic structures, where "(t, i, f)-neutrosophic" is referred to as refinement of the neutrosophic numerical components t, i, f.

Said Broumi and F. Smarandache published a paper [8] on refined neutrosophic numerical components in 2014.

8 Neutrosophic Hypercomplex Numbers of Dimension n

The *Hypercomplex Number of Dimension n* (or *n-Complex Number*) was defined by S. Olariu [10] as a number of the form:

$$u = x_o + h_1 x_1 + h_2 x_2 + \dots + h_{n-1} x_{n-1} \quad (10)$$

where $n \geq 2$, and the variables $x_0, x_1, x_2, \dots, x_{n-1}$ are real numbers, while $h_1, h_2, \dots, h_{n-1}$ are the complex units, $h_o = 1$, and they are multiplied as follows:

$$h_j h_k = h_{j+k} \text{ if } 0 \leq j+k \leq n-1, \text{ and } h_j h_k = h_{j+k-n} \text{ if } n \leq j+k \leq 2n-2. \quad (11)$$

We think that the above (11) complex unit multiplication formulas can be written in a simpler way as:

$$h_j h_k = h_{j+k \,(mod\, n)} \quad (12)$$

where *mod n* means *modulo n*.

For example, if $n = 5$, then $h_3 h_4 = h_{3+4(mod\,5)} = h_{7(mod5)} = h_2$.

Even more, formula (12) allows us to multiply many complex units at once, as follows:

$$h_{j1} h_{j2} \dots h_{jp} = h_{j1+j2+\dots+jp \,(mod\, n)}, \text{ for } p \geq 1. \quad (13)$$

We now define for the first time the *Neutrosophic Hypercomplex Number of Dimension n* (or *Neutrosophic n-Complex Number*), which is a number of the form:

$$u + vI, \quad (14)$$

where u and v are n-complex numbers and I = indeterminacy.

We also introduce now the *Refined Neutrosophic Hypercomplex Number of Dimension n* (or *Refined Neutrosophic n-Complex Number*) as a number of the form:

$$u + v_1 I_1 + v_2 I_2 + \dots + v_r I_r \quad (15)$$

where $u, v_1, v_2, \dots, v_r$ are n-complex numbers, and $I_1, I_2, \dots, I_r$ are sub-indeterminacies, for $r \geq 2$.

Combining these, we may define a *Hybrid Neutrosophic Hypercomplex Number* (or *Hybrid Neutrosophic n-Complex Number*), which is a number of the form $u + vI$, where either u or v is a n-complex number while the other one is different (may be an m-complex number, with $m \neq n$, or a real number, or another type of number).

And a *Hybrid Refined Neutrosophic Hypercomplex Number* (or *Hybrid Refined Neutrosophic n-Complex Number*), which is a number of the form $u + v_1 I_1 + v_2 I_2 + \dots + v_r I_r$, where at least one of $u, v_1, v_2, \dots, v_r$ is a n-complex number, while the others are different (may be m-complex numbers, with $m \neq n$, and/or a real numbers, and/or other types of numbers).

9 Neutrosophic Graphs

We now introduce for the first time the general definition of a *neutrosophic graph* [12], which is a (directed or undirected) graph that has some indeterminacy with respect to its edges, or with respect to its vertexes (nodes), or with respect to both (edges and vertexes simultaneously). We have four main categories of neutrosophic graphs:

1) The (t, i, f)-*Edge Neutrosophic Graph.*

In such a graph, the connection between two vertexes A and B, represented by edge AB:

has the neutroosphic value of (t, i, f).

2) *I-Edge Neutrosophic Graph.*

This one was introduced in 2003 in the book "Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps", by Dr. Vasantha Kandasamy and F. Smarandache, that used a different approach for the edge:

which can be just I = literal indeterminacy of the edge, with $I^2 = I$ (as in I-Neutrosophic algebraic structures). Therefore, simply we say that the connection between vertex A and vertex B is indeterminate.

3) *Orientation-Edge Neutrosophic Graph.*

At least one edge, let's say AB, has an unknown orientation (i.e. we do not know if it is from A to B, or from B to A).

4) *I-Vertex Neutrosophic Graph.*

Or at least one literal indeterminate vertex, meaning we do not know what this vertex represents.

5) (t, i, f)*-Vertex Neutrosophic Graph.*

We can also have at least one neutrosophic vertex, for example vertex A only partially belongs to the graph (t), indeterminate appurtenance to the graph (i), does not partially belong to the graph (f), we can say $A(t, i, f)$.

And combinations of any two, three, four, or five of the above five possibilities of neutrosophic graphs.

If (t, i, f) or the literal I are refined, we can get corresponding *refined neurosophic graphs*.

10 Example of Refined Indeterminacy and Multiplication Law of Sub-Indeterminacies

Discussing the development of Refined I-Neutrosophic Structures with Dr. W.B. Vasantha Kandasamy, Dr. A.A.A. Agboola, Mumtaz Ali, and Said Broumi, a question has arisen: if I is refined into $I_1, I_2, \dots, I_r$, with $r \geq 2$, how to define (or compute) $I_j * I_k$, for $j \neq k$?

We need to design a Sub-Indeterminacy $*$ Law Table.

Of course, this depends on the way one defines the algebraic binary multiplication law $*$ on the set:

$$\{N_r = a + b_1 I_1 + b_2 I_2 + \cdots + b_r I_r | a, b_1, b_2, \dots, b_r \in M\}, \quad (16)$$

where M can be $\mathbb{R}$ (the set of real numbers), or $\mathbb{C}$ (the set of complex numbers).

We present the below example.

But, first, let's present several (possible) interconnections between logic, set, and algebra.

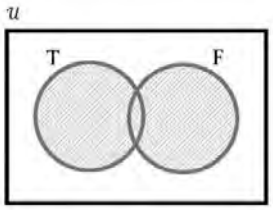
	Logic	*Set*	*Algebra*
operators	Disjunction (or) $\vee$	Union $\cup$	Addition $+$
	Conjunction (and) $\wedge$	Intersection $\cap$	Multiplication $\cdot$
	Negation $\neg$	Complement $\complement$	Subtraction $-$
	Implication $\rightarrow$	Inclusion $\subseteq$	Subtraction, Addition $-, +$
	Equivalence $\leftrightarrow$	Identity $\equiv$	Equality $=$

Table 1: Interconnections between logic, set, and algebra.

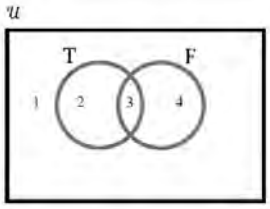
In general, if a Venn Diagram has n sets, with $n \geq 1$, the number of disjoint parts formed is 2^n. Then, if one combines the 2^n parts either by none, or by one, or by 2, ..., or by 2^n, one gets:

$$C_{2^n}^0 + C_{2^n}' + C_{2^n}^2 + \cdots + C_{2^n}^{2^n} = (1+1)^{2^n} = 2^{2^n}. \quad (17)$$

Hence, for $n = 2$, the Venn diagram, with literal truth



(T), and literal falsehood (F), will make $2^2 = 4$ disjoint parts, where the whole rectangle represents the whole universe of discourse $(\mathcal{U})$.

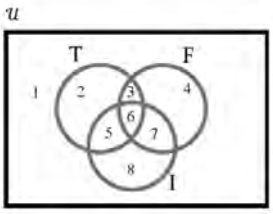


Venn Diagram for n =2.

Then, combining the four disjoint parts by none, by one, by two, by three, and by four, one gets

$$C_4^0 + C_4^1 + C_4^2 + C_4^3 + C_4^4 = (1+1)^4 = 2^4 = 16 = 2^{2^2}. \quad (18)$$

For $n = 3$, one has $2^3 = 8$ disjoint parts,

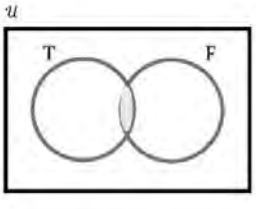


Venn Diagram for n = 3.

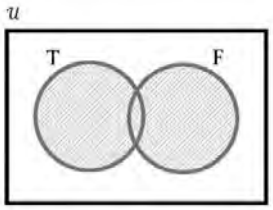
and combining them by none, by one, by two, and so on, by eight, one gets $2^8 = 256$, or $2^{2^3} = 256$.

For the case when $n = 2 = \{T, F\}$ one can make up to *16* sub-indeterminacies, such as:

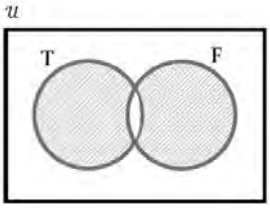
$$I_1 = C = \textbf{contradiction} = \text{True and False} = T \wedge F$$



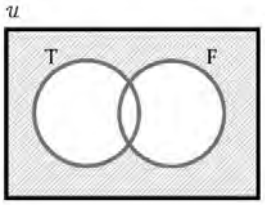
$I_2 = Y =$ **uncertainty** = True or False $= T \vee F$



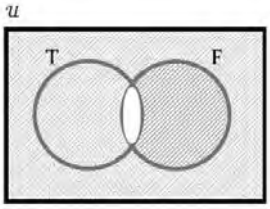
$I_3 = S =$ **unsureness** = either True or False $= T \veebar F$



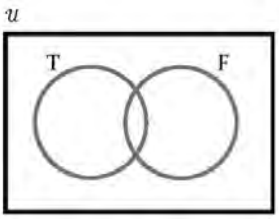
$I_4 = H =$ **nihilness** = neither True nor False
$= \neg T \wedge \neg F$



$I_5 = V =$ **vagueness** = not True or not False
$= \neg T \vee \neg F$



$I_6 = E =$ **emptiness** = neither True nor not True
$= \neg T \wedge \neg(\neg T) = \neg T \wedge T$



Let's consider the literal indeterminacy (I) refined into only six literal sub-indeterminacies as above.

The binary multiplication law

$$*: \{I_1, I_2, I_3, I_4, I_5, I_6\}^2 \to \{I_1, I_2, I_3, I_4, I_5, I_6\} \quad (19)$$

defined as:

$I_j * I_k$ = intersections of their Venn diagram representations;
or $I_j * I_k$ = application of $\wedge$ operator, i.e. $I_j \wedge I_k$.

We make the following:

$*$	I_1	I_2	I_3	I_4	I_5	I_6
I_1	I_1	I_1	I_6	I_6	I_6	I_6
I_2	I_1	I_2	I_3	I_6	I_3	I_6
I_3	I_6	I_3	I_3	I_6	I_3	I_6
I_4	I_6	I_6	I_6	I_4	I_4	I_6
I_5	I_6	I_3	I_3	I_4	I_5	I_6
I_6	I_6	I_6	I_6	I_6	I_6	I_6

Table 2: Sub-Indeterminacies Multiplication Law

11 Remark on the Variety of Sub-Indeterminacies Diagrams

One can construct in various ways the diagrams that represent the sub-indeterminacies and similarly one can define in many ways the $*$ algebraic multiplication law, $I_j * I_k$, depending on the problem or application to solve.

What we constructed above is just an example, not a general procedure.

Let's present below several calculations, so the reader gets familiar:

$I_1 * I_2 =$ (shaded area of I_1) $\cap$ (shaded area of I_2) = shaded area of I_1,
or $I_1 * I_2 = (T \wedge F) \wedge (T \vee F) = T \wedge F = I_1$.
$I_3 * I_4 =$ (shaded area of I_3) $\cap$ (shaded area of I_4) = empty set $= I_6$,
or $I_3 * I_4 = (T \veebar F) \wedge (\neg T \wedge \neg F) = [T \wedge (\neg T \wedge \neg F)] \veebar [F \wedge (\neg T \wedge \neg F)] = (T \wedge \neg T \wedge \neg F) \veebar (F \wedge \neg T \wedge \neg F) =$ (impossible) $\veebar$ (impossible)
because of $T \wedge \neg T$ in the first pair of parentheses and because of $F \wedge \neg F$ in the second pair of parentheses
$=$ (impossible) $= I_6$.
$I_5 * I_5 =$ (shaded area of I_5) $\cap$ (shaded area of I_5) = (shaded area of I_5) = I_5,
or $I_5 * I_5 = (\neg T \vee \neg F) \wedge (\neg T \vee \neg F) = \neg T \vee \neg F = I_5$.

Now we are able to build refined I-neutrosophic algebraic structures on the set

$$S_6 = \{a_0 + a_1 I_1 + a_2 I_2 + \cdots + a_6 I_6, \text{for } a_0, a_1, a_2, \ldots\ a_6 \in \mathbb{R}\}, \quad (20)$$

by defining the addition of refined I-neutrosophic numbers:

$$(a_0 + a_1 I_1 + a_2 I_2 + \cdots + a_6 I_6) + (b_0 + b_1 I_1 + b_2 I_2 + \cdots + b_6 I_6) = (a_0 + b_0) + (a_1 + b_1) I_1 + (a_2 + b_2) I_2 + \cdots + (a_6 + b_6) I_6 \in S_6. \quad (21)$$

And the multiplication of refined neutrosophic numbers:

$$(a_0 + a_1I_1 + a_2I_2 + \cdots + a_6I_6) \cdot (b_0 + b_1I_1 + b_2I_2 + \cdots + b_6I_6) = a_0b_0 + (a_0b_1 + a_1b_0)I_1 + (a_0b_2 + a_2b_0)I_2 + \cdots + (a_0b_6 + a_6b_0)I_6 + \\ + \sum_{j,k=1}^{6} a_jb_k\left(I_j * I_k\right) = a_0b_0 + \sum_{k=1}^{6}(a_0b_k + a_kb_0)I_k + \sum_{j,k=1}^{6} a_jb_k\left(I_j * I_k\right) \in S_6, \quad (22)$$

where the coefficients (scalars) $a_m \cdot b_n$, for $m = 0, 1, 2, \ldots, 6$ and $n = 0, 1, 2, \ldots, 6$, are multiplied as any real numbers, while $I_j * I_k$ are calculated according to the previous Sub-Indeterminacies Multiplication Law (Table 2).

Clearly, both operators (addition and multiplication of refined neutrosophic numbers) are well-defined on the set S_6.

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Neutrosophic Decision Making Model for Clay-Brick Selection in Construction Field Based on Grey Relational Analysis

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Abstract. The purpose of this paper is to present quality clay-brick selection approach based on multi-attribute decision making with single valued neutrosophic grey relational analysis. Brick plays a significant role in construction field. So it is important to select quality clay-brick for construction based on suitable mathematical decision making tool. There are several selection methods in the literature. Among them decision making with neutrosophic set is very pragmatic and interesting. Neutrosophic set is one tool that can deal with indeterminacy and inconsistent data. In the proposed method, the rating of all alternatives is expressed with single-valued neutrosophic set which is characterized by truth-membership degree (acceptance), indeterminacy membership degree and falsity membership degree (rejection). Weight of each attribute is determined based on experts' opinions. Neutrosophic grey relational coefficient is used based on Hamming distance between each alternative to ideal neutrosophic estimates reliability solution and ideal neutrosophic estimates unreliability solution. Then neutrosophic relational degree is used to determine the ranking order of all alternatives (bricks). An illustrative numerical example for quality brick selection is solved to show the effectiveness of the proposed method.

Keywords: Single-valued neutrosophic set; grey relational analysis; neutrosophic relative relational degree, multi-attribute decision making; clay-brick selection.

Introduction

Operations research management science has been mostly studied with structured and well defined problems with crisply or fuzzily defined information. However, in realistic decision making situations, some information cannot be defined crisply or fuzzily where indeterminacy involves. In order to deal with this situation neutrosophic set studied by Smarandache [15] is very helpful. Several researchers studied decision making problems [4, 19, 20] using single valued neutrosophic set proposed by Wang et al. [16]. Ye [18] proposed multi-criteria decision making problem using single valued neutrosophic sets. Biswas et al. [2] proposed multi attribute decision making (MADM) using neutrosophic grey relational analysis. Biswas et al.[3] also proposed a new method for MADM using entropy weight information based on neutrosophic grey relation analysis. Mondal and Pramanik [11] applied neutrosophic grey relational analysis based MADM to modeling school choice problem. Mondal and Pramanik [10] also applied single valued neutrosophic decision making concept for teacher recruitment in higher education. Mondal and Pramanik [9] also proposed a hybrid model namely rough neutrosophic multi-attribute decision making and applied in educational problem. So decision making in neutrosophic environment is an emergence area of research.

Brick selection is a special type of personnel selection problem. Pramanik and Mukhopadhyaya [12] studied grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Robertson and Smith [13] presented good reviews on personnel selection studies. They investigated the role of job analysis, contemporary models of work performance, and set of criteria employed in personnel selection process. Brick selection problem in intuitionistic fuzzy environment is studied by Mondal and Pramanik [8]. Brick selection problem in neutrosophic environment is yet to appear in the literature. In this paper brick selection in neutrosophic environment is studied.

Bricks are traditionally selected based on its color, size and total cost of brick, without considering the complexity

of indeterminacy involved in characterizing the attributes of brick. In that case the building construction may have some problems regarding low rigidity, longevity, etc, which cause great threat for the construction. However, indeterminacy inherently involves in some of the attributes of bricks. So it is necessary to formulate new scientific based selection method which is capable of handling indeterminacy related information. In order to select the most suitable brick to construct a building, the following criteria of bricks obtained from experts' opinions considered by Mondal and Pramanik [8] are used in this paper. The criteria are namely, solidity, color, size and shape, strength of brick, cost of brick, and carrying cost.

A good quality brick is characterized by its regular shape and size, with smooth even sides and no cracks or defects. Poor quality bricks are generally produced as a result of employing poor techniques but these errors can often be easily corrected. If bricks are well-made and well fired, a metallic sound or ring is heard when they are knocked together. If the produced sound is a dull sound, it reflects that the bricks are either cracked or under fired [1, 14]. In the proposed approach, the information provided by the experts about the attribute values assumes the form of single valued neutrosophic set. In the proposed approach, the ideal neutrosophic estimates reliability solution and the ideal neutrosophic estimate un-reliable solution are used. Neutrosophic grey relational coefficient of each alternative is determined to rank the alternatives.

Rest of the paper is organized in the following manner. Section 2 presents preliminaries of neutrosophic sets. Section 3 describes the attributes of brick and their operational definitions. Section 4 is devoted to present multi-attribute decision making based on neutrosophic grey relational analysis for brick selection process. In section 5, illustrative example is provided for brick selection process. Section 6 describes the advantage of the proposed approach. Section 7 presents conclusion and future direction of research work.

2 Mathematical Preliminaries

2.1 Neutrosophic Sets and single valued neutrosophic sets

Neutrosophic set is derived from neutrosophy, a new branch of philosophy studied by Smarandache [15]. Neutrosophy is devoted to study the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

2.1 Definition of Neutrosophic set [15]

Definition 1: Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is characterized by a truth membership function T_A an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A and F_A are real standard or non-standard subsets of $]^-0,1^+[$ that is $T_A: X \to]^-0,1^+[$; $I_A: X \to]^-0,1^+[$; $F_A: X \to]^-0,1^+[$ with the following relation

$$^-0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3^+, \quad \forall x \in X$$

Definition 2: The complement [15] of a neutrosophic set A is denoted by A^c and is defined by

$$T_{A^c}(x) = \{1^+\} - T_A(x) \quad ; \quad I_{A^c}(x) = \{1^+\} - I_A(x) \quad ; \quad F_{A^c}(x) = \{1^+\} - F_A(x)$$

Definition 3: (Containment [15]): A neutrosophic set A is contained in the other neutrosophic set B, denoted by $A \subseteq B$ if and only if the following result holds.

$$\inf T_A(x) \le \inf T_B(x) \ \sup T_A(x) \le \sup T_B(x)$$
$$\inf I_A(x) \ge \inf I_B(x), \ \sup I_A(x) \ge \sup I_B(x)$$
$$\inf F_A(x) \ge \inf F_B(x), \ \sup F_A(x) \ge \sup F_B(x)$$

for all x in X.

Definition 4: (SVNS) [16]: Let X be a universal space of points (objects), with a generic element of X denoted by x. A SVNS set S is characterized by a true membership function $T_S(x)$, a falsity membership function $I_S(x)$, and an indeterminacy function $F_S(x)$, with $T_S(x)$, $I_S(x)$, $F_S(x) \in [0, 1]$.

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in X$$

It should be noted that for a SVNS S,

$$0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3, \quad \forall x \in X$$

For example, suppose ten members of a school managing committee will critically review a specific agenda. Six of them agree with this agenda, three of them disagree and rest of one member remain undecided. Then by neutrosophic notation it can be expressed as $x\langle 0.6, 0.3, 0.1 \rangle$.

Definition 5: The complement of a SVNS S is denoted by S^c and is defined by

$$T_S{}^c(x) = F_S(x); \ I_S{}^c(x) = 1 - I_S(x); \ F_S{}^c(x) = T_S(x)$$

Definition 6: A SVNS S_A is contained in the other single valued neutrosophic set S_B, denoted as $S_A \subseteq S_B$ iff, $T_{S_A}(x) \le T_{S_B}(x)$; $I_{S_A}(x) \ge I_{S_B}(x)$; $F_{S_A}(x) \ge F_{S_B}(x)$, $\forall x \in X$.

Definition 7: Two single valued neutrosophic sets S_A and S_B are equal, i.e, $S_A = S_B$, if and only if

$S_A \subseteq S_B$ and $S_A \supseteq S_B$

Definition 8 (Union): The union of two SVNSs S_A and S_B is a SVNS S_C, written as

$S_C = S_A \cup S_B$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_A and S_B as follows:

$T_{SC}(x) = \max(T_{S_A}(x), T_{S_B}(x))$;

$I_{SC}(x) = \min(I_{S_A}(x), I_{S_B}(x))$;

$F_{SC}(x) = \min(F_{S_A}(x), F_{S_B}(x))$ for all x in X

Definition 9 (intersection): The intersection of two SVNSs S_A and S_B is a SVNS S_C written as $S_C = S_A \cap S_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_A and S_B as follows:

$T_{SC}(x) = \min(T_{S_A}(x), T_{S_B}(x))$;

$I_{SC}(x) = \max(I_{S_A}(x), I_{S_B}(x))$;

$F_{SC}(x) = \max(F_{S_A}(x), F_{S_B}(x)), \forall x \in X$

Distance between two neutrosophic sets

The general SVNS has the following pattern:

$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in X\}$

For finite SVNSs can be represented by the ordered tetrads:

$$S = \{(x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, (x_m/(T_S(x_m), I_S(x_m), F_S(x_m)))\}, \quad \forall x \in X$$

Definition 10: Let

$$S_A = \{(x_1/(T_{S_A}(x_1), I_{S_A}(x_1), F_{S_A}(x_1))), \cdots, (x_n/(T_{S_A}(x_n), I_{S_A}(x_n), F_{S_A}(x_n)))\}$$

$$S_B = \{(x_1/(T_{S_B}(x_1), I_{S_B}(x_1), F_{S_B}(x_1))), \cdots, (x_n/(T_{S_B}(x_n), I_{S_B}(x_n), F_{S_B}(x_n)))\}$$

be two single-valued neutrosophic sets

(SVNSs) in $x=\{x_1, x_2, x_3, ..., x_n\}$

Then the Hamming distance between two SVNSs S_A and S_B

is defined as follows:

$$d_S(S_A, S_B) = \sum_{i=1}^{n} \left\langle \begin{array}{l} |T_{S_A}(x) - T_{S_B}(x)| \\ + |I_{S_A}(x) - I_{S_B}(x)| \\ + |F_{S_A}(x) - F_{S_B}(x)| \end{array} \right\rangle \quad (1)$$

and normalized Hamming distance between two SVNSs S_A and S_B is defined as follows:

$$^{N}d_S(S_A, S_B) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{array}{l} |T_{S_A}(x) - T_{S_B}(x)| \\ + |I_{S_A}(x) - I_{S_B}(x)| \\ + |F_{S_A}(x) - F_{S_B}(x)| \end{array} \right\rangle \quad (2)$$

with the following two properties

1. $0 \le d_S(S_A, S_B) \le 3n$ (3)
2. $0 \le {}^{N}d_S(S_A, S_B) \le 1$ (4)

Definition 11: Ideal neutrosophic reliability solution INERS [18]

$Q_S^+ = \langle q_{S_1}^+, q_{S_2}^+, \cdots, q_{S_n}^+ \rangle$ is a solution in which every component is presented by $q_{S_j}^+ = \langle T_j^+, I_j^+, F_j^+ \rangle$ where $T_j^+ = \max_i \{T_{ij}\}$, $I_j^+ = \min_i \{I_{ij}\}$ and $F_j^+ = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$ for i = 1, 2, …, m, j = 1, 2, …, n

Definition 12: Ideal neutrosophic estimates unreliability solution (INEURS) [18]

$Q_S^- = \langle q_{S_1}^-, q_{S_2}^-, \cdots, q_{S_n}^- \rangle$ is a solution in which every component is represented by $q_{S_j}^- = \langle T_j^-, I_j^-, F_j^- \rangle$ where $T_j^- = \min_i \{T_{ij}\}$, $I_j^- = \max_i \{I_{ij}\}$ and $F_j^- = \max_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$ for i = 1, 2, …, m, j = 1, 2, …, n

3. Brick Attributes [8]

Six criteria [8] of bricks are considered, namely, solidity (C_1), color (C_2), size and shape (C_3), and strength of brick (C_4), brick cost (C_5), carrying cost (C_6). These six criteria's are explained as follows:

(i) Solid clay brick (C_1)**:** An ideal extended solid rigid body prepared by loam soil having fixed size and shape remains unaltered when fixed forces are applied. The distance between any two given points of the rigid body remains unchanged when external fixed forces applied on it. If we soap a solid brick in water and drop it from 3 or 4 feet heights [1, 14], it remains unbroken.

(ii) Color (C_2): Color of quality brick refers to reddish or light maroon.

(iii) Size and shape (C_3): All bricks are to be more or less same size and shape having same length, width and height. The size or dimensions of a brick are determined by how it is used in construction work. Standard size of a brick may vary. Size of a brick may be around 190mm × 90mm × 40mm [14].

Width

The width of a brick should be small enough to allow a bricklayer to lift the brick with one hand and place it on a bed of mortar. For the average person, the width should not be more than 115 mm.

Length

The length of a brick refers to twice its width plus 10 mm (for the mortar joint). A brick with this length will be easier to build with because it will provide and even surface on both sides of the wall. For example, if you follow the rule of the length being twice the width plus 10 mm, if you would like to have a brick x mm wide, then the ideal length would be (2x + 10) mm.

Height

The height of a brick is related to the length of the brick. The height of three bricks plus two 10 mm joints is equal to the length of a brick. This allows a bricklayer to lay bricks on end (called a soldier course) and join them into the wall without having to cut the bricks. The height of a brick is determined by subtracting 20 mm (the thickness of the two 10 mm mortar joints) from the length and dividing the result by three (this represents the three bricks).

Possible brick Sizes

In India the standard brick size is 190 mm x 90 mm x 40 mm while the British standard is 215 mm x 102.5 mm x 65 mm. To select your brick size, first contact the local public works department to see if your country has a standard size. If not, you will have to choose your own size. Possible brick sizes can be found in [8].

(iv) Well dried and burnt (strength of brick) (C_4) [14]: Raw bricks are well dried in sunshine and then properly burnt. If bricks have been well- made and well-fired, a metallic sound is heard when they are knocked together. If knocking creates a dull sound, it reflects that they are either cracked or under-fired. A simple test for strength of a brick is to drop it from a height of 1.2 meters (shoulder height). A good brick will not break. This test should be repeated with a wet brick (a brick soaked in water for one week). If the soaked brick does not break when dropped, it reflects that the quality of the brick is good enough to build single storied structures.

v) Brick cost (C_5): Decision maker always tries to minimize purchasing cost. Reasonable price of quality brick is more acceptable.

vi) Carrying cost (C_6): The distance between brick field and construction site must be reasonable for maintaining minimum carrying cost.

4. GRA method for multiple attribute decision making problems with single valued neutrosophic information

Consider a multi-attribute decision making problem with m alternatives and n attributes. Let $A_1, A_2, \ldots, A_m$ and $C_1, C_2, \ldots, C_n$ represent the alternatives and attributes respectively. The rating reflects the performance of the alternative A_i against the attribute C_j. For MADM, weight vector $W = w_1, w_2, \ldots, w_n$ is fixed to the attributes. The weight $w_j > 0$, j = 1, 2, 3,…, n reflects the relative importance of attributes C_j, j = 1, 2, …, n to the decision making process. The weights of the attributes are usually determined on subjective basis. The values associated with the alternatives for MADM problems presented in the decision table 1.

Table1: Decision table of attribute values

	C_1	C_2	…	C_n
A_1	d_{11}	d_{12}	…	d_{1n}
A_2	d_{21}	d_{22}	…	d_{2n}
.	…	…	…	…
.	…	…	…	…
A_m	d_{m1}	d_{m2}	…	d_{mn}

$$D = \langle d_{ij} \rangle_{m \times n} = \tag{5}$$

GRA is one of the derived evaluation methods for MADM based on the concept of grey relational space. The main procedure of GRA method is firstly translating the performance of all alternatives into a comparability sequence. According to these sequences, a reference sequence (ideal target sequence) is defined. Then, the grey relational coefficient between all comparability sequences and the reference sequence for different values of distinguishing coefficient are calculated. Finally, based on these grey relational coefficients, the grey relational degree between the reference sequence and every comparability sequences is calculated. If an alternative gets the highest grey relational grade with the reference sequence, it means that the comparability sequence is the most similar to the reference sequence and that alternative would be the best choice (Fung [7]). The steps of improved GRA under SVNS are described below:

Step 1. Determination of the most important criteria

Generally, there exist many criteria or attributes in decision making problems where some of them are important and others may not be so important. So it is important to select the proper criteria or attributes for decision making situations. The most important criterion may be selected based on experts' opinions.

Step 2. Construction of the decision matrix with single valued neutrosophic sets (SVNSs)

The rating of alternatives A_i (i = 1, 2, . . , m) with respect to the attribute C_j (j = 1, 2, . . ., n) is assumed as SVNS. It can be represented with the following forms:

$$A_i = \left[\frac{C_1}{\langle T_{i1}, I_{i1}, F_{i1} \rangle}, \frac{C_2}{\langle T_{i2}, I_{i2}, F_{i2} \rangle}, \ldots, \frac{C_n}{\langle T_{in}, I_{in}, F_{in} \rangle} \right]; \; C_j \in C$$

$$= \left[\frac{C_j}{\langle T_{ij}, I_{ij}, F_{ij} \rangle} : C_j \in C \right] \text{ for j = 1, 2,…, n}$$

Here T_{ij}, I_{ij}, F_{ij} are the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i is satisfying the attribute C_j, respectively where

$$0 \le T_{ij} \le 1, \quad 0 \le I_{ij} \le 1, \quad 0 \le F_{ij} \le 1 \; \textit{and} \; 0 \le T_{ij} + I_{ij} + F_{ij} \le 3$$

The decision matrix D_S is presented in the table 2.

Table2. Decision matrix D_S

$$D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n} =$$

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11} \rangle$	$\langle T_{12}, I_{12}, F_{12} \rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n} \rangle$
A_2	$\langle T_{21}, I_{21}, F_{21} \rangle$	$\langle T_{22}, I_{22}, F_{22} \rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{m1} \rangle$	$\langle T_{m2}, I_{m2}, F_{m2} \rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn} \rangle$

(6)

Step 3. Determination of the weights of criteria

In the decision making process, decision maker may often encounter with unknown attribute weights. It may happen that the importance of the attributes is different. Therefore we need to determine reasonable attribute weight for making a proper decision.

Step 4. Determination of the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS) for neutrosophic decision matrix.

For a neutrosophic decision making matrix $D_S = \lfloor q_{S_{ij}} \rfloor_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$, T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i of A satisfying the attribute C_j of C. The neutrosophic estimate reliability solution can be determined from the concept of SVNS cube proposed by Dezert [5]

Step 5. Calculation of the neutrosophic grey relational coefficient of alternative from INERS

Grey relational coefficient of each alternative from INERS is as follows:

$$g_{ij}^{+} = \frac{\min\limits_i \min\limits_j \Delta_{ij}^{+} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{+}}{\Delta_{ij}^{+} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{+}}, \text{ where}$$

$$\Delta_{ij}^{+} = d\left(q_{S_j}^{+}, q_{S_{ij}}\right) \quad (7)$$

i = 1, 2, …, m and j = 1, 2, …., n

Step 6. Calculation of the neutrosophic grey relational coefficient of alternative from INEURS

Grey relational coefficient of each alternative from INEURS is as follows:

$$g_{ij}^{-} = \frac{\min\limits_i \min\limits_j \Delta_{ij}^{-} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{-}}{\Delta_{ij}^{-} + \rho \max\limits_i \max\limits_j \Delta_{ij}^{-}}, \text{ where}$$

$$\Delta_{ij}^{-} = d\left(q_{S_{ij}}, q_{S_j}^{-}\right) \quad (8)$$

i = 1, 2, …, m and j = 1, 2, …., n

$\rho \in [0,1]$ is the distinguishable coefficient or the identification coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho = 1$, the comparison environment is unaltered; when $\rho = 0$, the comparison environment disappears. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, $\rho = 0.5$ is considered for decision making.

Step 7. Calculation of the neutrosophic grey relational coefficient

Calculate the degree of neutrosophic grey relational coefficient of each alternative from INERS and INEURS using the following equation respectively:

$$g_i^{+} = \sum_{j=1}^{n} w_j g_{ij}^{+} \text{ for i = 1, 2, …, m} \quad (9)$$

$$g_i^{-} = \sum_{j=1}^{n} w_j g_{ij}^{-} \text{ for i = 1, 2, …, m} \quad (10)$$

Step 8. Calculation the neutrosophic relative relational degree

We calculate the neutrosophic relative relational degree of each alternative from indeterminacy truthfulness falsity positive ideal solution (ITFPIS) with the help of following equations:

$$R_i = \frac{g_i^{+}}{g_i^{-} + g_i^{+}}, \text{ for i = 1, 2, …, m} \quad (11)$$

Step 9. Ranking the alternatives

According to the relative relational degree, the ranking order of all alternatives can be determined. The highest value of R_i represents the most important alternative.

Step10. End

5. Example of Brick selection

The steps of brick selection procedure using the proposed approach are arranged as follows:

Step 1: Determination of the most important criteria

The most important criterion of brick is selected based on experts' opinions are namely, solidity, color, size and shape, strength of brick, cost of brick, and carrying cost.

Step 2: Construction of the decision matrix with single valued neutrosophic sets (SVNSs)

Here the most important criterion of brick is chosen based on experts' opinions. When the four possible alternatives with respect to the six criteria are evaluated by the expert, we can obtain the following single-valued neutrosophic decision matrix:

$D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{4\times 6} =$

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	$\langle 0.7,0.2,0.1\rangle$	$\langle 0.8,0.1,0.1\rangle$	$\langle 0.7,0.1,0.2\rangle$	$\langle 0.6,0.2,0.2\rangle$	$\langle 0.7,0.0,0.0\rangle$	$\langle 0.5,0.3,0.3\rangle$
A_2	$\langle 0.7,0.1,0.0\rangle$	$\langle 0.7,0.2,0.1\rangle$	$\langle 0.8,0.1,0.1\rangle$	$\langle 0.6,0.1,0.1\rangle$	$\langle 0.8,0.1,0.1\rangle$	$\langle 0.5,0.4,0.1\rangle$
A_3	$\langle 0.8,0.1,0.1\rangle$	$\langle 0.8,0.0,0.1\rangle$	$\langle 0.7,0.2,0.1\rangle$	$\langle 0.6,0.2,0.1\rangle$	$\langle 0.7,0.1,0.1\rangle$	$\langle 0.6,0.4,0.2\rangle$
A_4	$\langle 0.7,0.0,0.1\rangle$	$\langle 0.9,0.2,0.0\rangle$	$\langle 0.7,0.3,0.1\rangle$	$\langle 0.8,0.2,0.1\rangle$	$\langle 0.6,0.2,0.3\rangle$	$\langle 0.7,0.3,0.2\rangle$

(12)

Step 3. Determination weights of the criteria

In the decision making situation, decision makers recognize that all the criteria of bricks are not equal importance. Here the importance of the criteria is obtained from expert opinion through questionnaire method i.e. the weights of the criteria are previously determined such that the sum of the weights of the criteria is equal to unity. Data was collected from fifteen constructional engineers, ten construction labors of Nadia district from twelve brick fields of surrounding areas. After extended interviews and discussions with the experts, the criteria of brick were found the same as found in [8] namely, solidity, color, size and shape, strength of brick, brick cost, and carrying cost. We have the weight of each criterion w_j, j = 1, 2, 3, 4, 5, 6 as follows:

$w_1 = 0.275$, $w_2 = 0.175$, $w_3 = 0.2$, $w_4 = 0.1$, $w_5 = 0.05$, $w_6 = 0.2$ such that $\sum_{j=1}^{6} w_j = 1$

Step 4. Determine the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS)

$$Q_S^+ = \left[q_{S_1}^+, q_{S_2}^+, q_{S_3}^+, q_{S_4}^+, q_{S_5}^+, q_{S_6}^+\right] =$$

$$\begin{bmatrix} \left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\}\right\rangle, \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\}\right\rangle, \\ \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\}\right\rangle, \left\langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\}\right\rangle, \\ \left\langle \max_i\{T_{i5}\}, \min_i\{I_{i5}\}, \min_i\{F_{i5}\}\right\rangle, \left\langle \max_i\{T_{i6}\}, \min_i\{I_{i6}\}, \min_i\{F_{i6}\}\right\rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.8,0.0,0.0\rangle, \langle 0.9,0.0,0.0\rangle, \langle 0.8,0.1,0.1\rangle, \\ \langle 0.6,0.1,0.1\rangle, \langle 0.8,0.0,0.0\rangle, \langle 0.7,0.3,0.1\rangle \end{bmatrix}$$

$$Q_S^- = \left[q_{S_1}^-, q_{S_2}^-, q_{S_3}^-, q_{S_4}^-, q_{S5}^-, q_{S_6}^-\right] =$$

$$\begin{bmatrix} \left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\}\right\rangle, \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\}\right\rangle, \\ \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\}\right\rangle, \left\langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\}\right\rangle, \\ \left\langle \min_i\{T_{i5}\}, \max_i\{I_{i5}\}, \max_i\{F_{i5}\}\right\rangle, \left\langle \min_i\{T_{i6}\}, \max_i\{I_{i6}\}, \max_i\{F_{i6}\}\right\rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle 0.7,0.2,0.1\rangle, \langle 0.7,0.2,0.1\rangle, \langle 0.7,0.3,0.2\rangle, \\ \langle 0.6,0.2,0.2\rangle, \langle 0.6,0.2,0.3\rangle, \langle 0.5,0.4,0.3\rangle \end{bmatrix}$$

Step 5. Calculation of the neutrosophic grey relational coefficient of each alternative from INERS

Using Equation (7), the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as follows:

$$g_{ij}^+ = \begin{bmatrix} 0.3333 & 0.4641 & 0.4641 & 0.4641 & 1.0000 & 0.4545 \\ 0.4641 & 0.2899 & 1.0000 & 1.0000 & 0.7143 & 0.5556 \\ 0.4641 & 0.4641 & 0.4641 & 0.5505 & 0.5556 & 0.5556 \\ 0.4641 & 0.3797 & 0.3539 & 0.3539 & 0.5556 & 1.0000 \end{bmatrix} \quad (13)$$

Step 6. Calculation of the neutrosophic grey relational coefficient of each alternative from INEURS

Similarly, from Equation (8) the neutrosophic grey relational coefficient of each alternative from INEURS can be obtained as follows:

$$g_{ij}^- = \begin{bmatrix} 1.0000 & 0.4641 & 0.3797 & 1.0000 & 0.3333 & 1.0000 \\ 0.4641 & 1.0000 & 0.3333 & 0.4641 & 0.3750 & 0.7500 \\ 0.4641 & 0.3539 & 0.4641 & 0.5505 & 0.4286 & 0.7500 \\ 0.3797 & 0.3539 & 0.5505 & 0.3539 & 1.0000 & 0.5000 \end{bmatrix} \quad (14)$$

Step 7. Determination of the degree of neutrosophic grey relational co-efficient of each alternative from INERS and INEURS

The required neutrosophic grey relational co-efficient corresponding to INERS is obtained using equation (9) as follows:

$$g_1^+ = 0.63635,\ g_2^+ = 0.62520, g_3^+ = 0.49562,\ g_4^+ = 0.52720 \quad (15)$$

and corresponding to INEURS is obtained with the help of equation (10) as follows:

$$g_1^- = 0.74882,\ g_2^- = 0.58445, g_3^- = 0.50886,\ g_4^- = 0.46184 \quad (16)$$

Step 8. Calculation of neutrosophic relative relational degree

Thus neutrosophic relative degree of each alternative from INERS can be obtained with the help of equation (11) as follows:

$R_1 = 0.459402;\ R_2 = 0.516844;$
$R_3 = 0.493410;\ R_4 = 0.533042$ (17)

Step 9. Ranking the alternatives

The ranking order of all alternatives can be determined according the value of neutrosophic relational degree i.e.

$R_4 > R_2 > R_3 > R_1$

It is seen that the highest value of neutrosophic relational degree is R_4. Therefore the best alternative brick is identified as A_4.

Step10. End

6. Advantages of the proposed approach

The proposed approach is very flexible as it uses the realistic nature of attributes i.e. the degree of indeterminacy as well as degree of rejection and acceptance simultaneously. In this paper, we showed how the proposed approach could provide a well-structured, practical, and scientific selection. New criteria are easily incorporated in the formulation of the proposed approach.

Conclusion

In the study, the concept of single valued neutrosophic set proposed by Wang et al. [16] with grey relational analysis [6] is used to deal with realistic brick selection process. Neutrosophic decision making based on grey relational analysis approach is a practical, versatile and powerful tool that identifies the criteria and offers a consistent structure and process for selecting bricks by employing the concept of acceptance, indeterminacy and rejection of single valued neutrosophic sets simultaneously. In the study, we demonstrated how the proposed approach could provide a well-structured, rational, and scientific selection practice.

Therefore, in future, the proposed approach can be used for dealing with multi-attribute decision-making problems such as project evaluation, supplier selection, manufacturing system, data mining, medical diagnosis and many other areas of management decision making. Neutrosophic sets, degree of rejection (non membership), degree of acceptance (membership) and degree of indeterminacy (hesitancy) are independent to each other. In this sense, the concept of single valued neutrosophic set applied in this paper is a realistic application of brick selection process. This selection process can be extended to the environment dealing with interval single valued neutrosophic set [17].

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Neutrosophy-based Interpretation of Four Mechanical Worldviews

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Abstract. We take into consideration four mechanical worldviews: the single "gravity" (as *<A>,* or *t*); the single "repulsion" (as *<antiA>,* or *f*); the "gravity and repulsion" taking the same object as carrier (as *<neutA>*$_1$, or I_1 = first type of subindeterminacy); and the contradictory objects of "gravity and repulsion" formed by natural external force and natural repulsive force (as *<neutA>*$_2$, or I_2 = second type of subindeterminacy), and interpret them by employing Neutrosophy, expanding their application scope. We point out that the fourth mechanical worldview is consistent with the Neutrosophic tetrad, and that the natural external force and natural repulsive force are the correct qualitative analysis of the natural forces of the universe, and thus it can be used to interpret a variety of phenomena of the universe.

Keywords: Mechanical worldview, Neutrosophy, Neutrosophic tetrad, natural force of the universe, natural external force, natural repulsive force.

1 Introduction

Various mechanical worldviews were proposed through the ages. We select four mechanical worldviews to be interpreted through neutrosophic theory and method.

Neutrosophy, introduced by Prof. Florentin Smarandache in 1995, is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas are both referred to as <Non-A>.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics, which are used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic (NL) is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each space dimension represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[without a necessary connection between them.

For example, we talk about two opposite forces, gravity and repulsion, but also the "gravity and repulsion together" (= indeterminacy, from neutrosophy). Then, we also split this indeterminacy into two indeterminacies (see reference [3]) such as:

I_1 = gravity and repulsion with the same carrier, and

I_2 = gravity and repulsion without the same carrier.

Smarandache introduced the degree of indeterminacy/ neutrality (i) as independent component, and defined the neutrosophic set, coining the words “neutrosophy” and “neutrosophic”. In 2013, he refined/split the neutrosophic set to n components: $t_1, t_2, \ldots t_j$; $i_1, i_2, \ldots, i_k$; $f_1, f_2, \ldots, f_l$, with $j+k+l = n > 3$. In our article we have *t, i_1, i_2,* and *f.* Hence indeterminacy *i* was split into two subindeterminacies i_1 and i_2.

More about Neutrosophy can be found in references [1-3].

Obviously, we have broad application prospects when combining various mechanical worldviews with Neutrosophy.

We now interpret the four mechanical worldviews through Neutrosophy, for issues related to the effect of "gravity" and "repulsion", and the like.

2 Isaac Newton (English scholar): the mechanical worldview of single "gravity"

Newton's "law of gravity" is the beginning of the mechanical worldview of single "gravity". But natheless, it can only be used to describe the attraction between objects, and not the repulsion caused by light and heat radiation of celestial objects, therefore it cannot be used to solve the issues of mutual repulsion and departure between celestial objects. In addition, the mechanical worldview of single "gravity" does not explain the celestial objects' lateral motion and gravitational instantaneous transmitting, and it does not reveal the nature of gravity. So, in many ways, Newton's "gravity" is erroneous and one-sided, and it can not be taken as an accurate qualitative analysis of the natural forces of the universe.

However, according to the neutrosophic viewpoint, any theory, or law, holds three situations: truth, falsehood, and indeterminacy. Because the law of gravity applies for some issues, we conclude, after interpreting it through Neutrosophy, that it is conducive to further development and improvement.

3 Edwin P. Hubble (American scholar): the mechanical worldview of single "repulsion"

The mechanical worldview of single "repulsion" is derived by Hubble according to the galaxy redshift, the expansion of the universe, and the light and heat radiation of celestial objects, in order to solve the issues of the mutual repulsion and departure between them. While not only this view cannot be used to solve the problems of celestial objects' lateral motion, but also cannot answer the reason that the apple should indeed go down to the land, therefore, in general, the mechanical worldview of single "repulsion" is fallacious. Consequently, the "Hubble's law" also needs to be further developed and improved.

4 Albert Einstein (American scholar): the mechanical worldview of the "gravity and repulsion" taking the same object as the carrier

Adding to gravity and repulsion the cosmological constant (repulsion) on a "gravitational field equation" of space-time warpage, Einstein built another mechanical worldview. Later on, Einstein found that the universe devised by gravity and repulsion is a static one, and thus not meeting the dynamical universe as the observed data. Thereby, the cosmological constant (repulsion) was abandoned. Actually, this view was not firstly proposed by Einstein: before that, Kant, Hegel, Marx, Engels, Lenin, and others, have already enunciated similar viewpoints of "attraction and repulsion". Even if Einstein's cosmological constant (repulsion) was considered to be accurate for a long time, in modern physics the mechanical worldview of the "gravity and repulsion" was rebuilt, disposing "gravity" to explain the mutual attraction between objects, and "repulsion" to define the mutual exclusion between objects. But that's just subjective wishful thinking. Because this mechanical worldview takes the same object as the carrier, according to a philosophical judgement criterion, the "gravity and repulsion" do not constitute two contradictory objects, therefore we face a fabricated and false concept. To sum up, using the primary and the secondary contradictions in modern physics to discuss the mutual transformation of the "gravity and repulsion" on the same celestial object is misleading, and the mechanical worldview of the "gravity and repulsion" taking the same object as the carrier is erroneous.

5 Luo Zhengda (Chinese scholar): the mechanical worldview of natural external force and natural repulsive force

This mechanical worldview is fundamentally different from Einstein's view. Taking natural external force and natural repulsive force as the core, this mechanical worldview points out that the natural external force is the energy field of the universal space, having contraction and aggregation as natural property. The mutual aggregation of celestial objects is not the mutual attraction, but the mutual aggregation follows the contraction and aggregation of the energy field.

Enacted by the natural property of contraction and aggregation of natural external force, the energy of celestial objects' core accumulates. The mass can also be changed into energy, thus creating the radiation of repulsion, and forming the celestial objects' repulsion field taking celestial objects' core as the center. The mutual opposition and rejection between celestial objects are caused by the actions of contraction and aggregation of natural external force.

Essentially, all natural external forces and natural repulsive forces are energy matter. Natural external force is caused by aggregation of celestial objects, and natural repulsive force is provoked by repulsion of celestial objects. Natural external force is taking the space energy field as the carrier, and natural repulsive force is taking the celestial object of mass as the carrier. The two carriers of natural external force and natural repulsive force are different and contradictory, and thus meet the philosophical condition of contradiction.

This mechanical worldview is consistent with the principle and analysis of Neutrosophy.

In reference [2], the dialectical triad thesis-antithesis-synthesis of Hegel is extended to the neutrosophic tetrad thesis-antithesis-neutrothesis-neutrosynthesis.

A neutrosophic synthesis (neutrosynthesis) is more refined that the dialectical synthesis. It carries on the unification and synthesis regarding the opposites and their neutrals too.

A neutrosophic synthesis (neutrosynthesis) includes <A>, <Anti-A>, <Neut-A>, <A> plus <Anti-A>, <A> plus <Neut-A>, <Anti-A> plus <Neut-A>, <A> plus <Anti-A> plus <Neut-A>, and so on.

Similarly, the mechanical worldview taking natural external force and natural repulsive force as the core also considers and includes various situations and combinations related to "gravity", "repulsion", and the like.

Therefore, natural external force and natural repulsive force are correct qualitative analysis of the natural force of the universe, thus this mechanical worldview is flawless and can be used to interpret a variety of phenomena of the universe. Detailed information can be found in references [4-8], where Luo Zhengda coined the concepts of “natural force of the universe”,“natural external force”, and “natural repulsive force”.

6 Conclusions

Natural external force causes the aggregation of celes-

tial objects, while natural repulsive force causes the repulsion of celestial objects. By combining the theory of the natural force of the universe with Neutrosophy, we conclude that the two theories complement each other, creating new paths for interpreting and dealing with all sort of natural phenomena.

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Application of Neutrosophic Set Theory in Generalized Assignment Problem

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Abstract. This paper presents the application of Neutrosophic Set Theory (NST) in solving Generalized Assignment Problem (GAP). GAP has been solved earlier under fuzzy environment. NST is a generalization of the concept of classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set. Elements of Neutrosophic set are characterized by a truth-membership function, falsity and also indeterminacy which is a more realistic way of expressing the parameters in real life problem. Here the elements of the cost matrix for the GAP are considered as neutrosophic elements which have not been considered earlier by any other author. The problem has been solved by evaluating score function matrix and then solving it by Extremum Difference Method (EDM) [1] to get the optimal assignment. The method has been demonstrated by a suitable numerical example.

Keywords: NST, GAP, EDM.

1. Introduction

The concept of fuzzy sets and the degree of membership/truth (T) was first introduced by Zadeh in 1965 [2]. This concept is very much useful to handle uncertainty in real life situation. After two decades, Turksen [3] introduced the concept of interval valued fuzzy set which was not enough to consider the non-membership function. In the year 1999, Atanassov [4], [5], [6] proposed the degree of nonmembership/falsehood (F) and intuitionistic fuzzy set (IFS) which is not only more practical in real life but also the generalization of fuzzy set. The paper considers both the degree of membership $\mu_A(x)$ $\in$ [0, 1] of each element x $\in$ X to a set A and the degree of non-membership $\nu_A(x)$ $\in$ [0, 1] s.t. $\mu_A(x) + \nu_A(x) \leq 1$. IFS deals with incomplete information both for membership and non-membership function but not with indeterminacy membership function which is also very natural and obvious part in real life situation. Wang et. Al [7] first considered this indeterminate information which is more practical and useful in real life problems. F.Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set. He coined the words "neutrosophy" and "neutrosophic". In 2013 he refined the neutrosophic set to n components: t1, t2, ...; i1, i2, ...; f1, f2, ... So in this paper we have used the neutrosophic set theory to solve GAP which hasn't been done till now.

2. Preliminaries

2.1 Neutrosophic Set [8]

Let U be the space of points (or objects) with generic element 'x'. A neutrosophic set A in U is characterized by a truth membership function T_A, and indeterminacy function I_A and a falsity membership function F_A, where T_A, I_A and F_A are real standard or non-standard subsets of $]^-0, 1^+[$, i.e sup T_A: $x \rightarrow]^-0, 1^+[$

sup F_A: $x \rightarrow]^-0, 1^+[$

$$\sup I_A: x \rightarrow]^-0, 1^+[$$

A neutrosophic set A upon U as an object is defined as

$$\frac{x}{T_A(x), I_A(x), F_A(x)} = \{ \frac{x}{T_A, I_A, F_A} : x \in U \},$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are subintervals or union of subintervals of [0, 1].

2.2 Algebraic Operations with Neutrosophic Set [8]

For two neutrosophic sets A and B where and

a> Complement of A
$A^{/} = \{ \frac{x}{T,I,F} \mid T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \}$

b> Intersection of A and B
$A \cap B = \{ \frac{x}{T,I,F} \mid T = T_A T_B, I = I_A I_B, F = F_A F_B \}$

c> Union of A and B
$A \cup B = \{ \frac{x}{T,I,F} \mid T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \}$

d> Cartesian Product of A and B

$$A \times B = \{ (\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B}) \mid \frac{x}{T_A, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \}$$

e> A is a subset of B

$$A \subseteq B \; \forall \; \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B, T_A \leq T_B, I_A \geq I_B \text{ and } F_A \geq F_B$$

f> Difference of A and B
$A \backslash B = \{ \frac{x}{T,I,F} \mid T = T_A - T_A T_B, I = I_A - I_A I_B, F = F_A - F_A F_B \}$

NST can be used in assignment problem (AP) and Generalized Assignment Problem (GAP).

3. Generalized Assignment Problem using Neutrosophic Set Theory

In this section, we have formulated the GAP using NST. GAP has been solved earlier in different ways by different mathematicians. David B. Shymos and Eva Tardos [April, 1991] considered the GAP as the problem of scheduling parallel machines and solved it by polynomial time algorithm. Dr. Zeev Nutov [June, 2005] solved GAP considering it as a Max-profit scheduling problems. Supriya Kar, Dr. Kajla Basu, Dr. Sathi Mukherjee [International Journal of Fuzzy Mathematics & Systems, Vol.4, No.2 (2014) pp.-169-180] solved GAP under fuzzy environment using Extremum Difference Method. Supriya Kar et. al solved FGAP with restriction on the cost of both job & person using EDM [International Journal of management, Vol.4, Issue5, September-October (2013) pp.-50-59]. They solved FGAP also under Hesitant Fuzzy Environment [Springer India, Opsearch, 29th October 2014, ISSN 0030-3887.

Here, we have used NST to solve GAP because in neutrosophy, every object has not only a certain degree of truth, but also a falsity degree and an indeterminacy degree that have to be considered independently.

3.1 Mathematical model for GAP under Neutrosophic Set Theory

Let us consider a GAP under neutrosophic set in which there are m jobs J= {J1, J2,……,J_m} and n persons P= { P_1,P_2,……..,P_n}. The cost matrix of the Neutrosophic Generalized Assignment Problem (NGAP) contains neutrosophic elements denoting time for completing j-th job by i-th machine and the mathematical model for NGAP will be as follows-

Model 1.

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij} \quad \ldots\ldots\ldots [1]$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1, \; i = 1, 2, \ldots\ldots, m \quad \ldots\ldots\ldots [2]$$

$$\sum_{i=1}^{m} c_{ij}x_{ij} \leq a_j, \; j = 1,2,\ldots\ldots,n \quad \ldots\ldots [3]$$

X_{ij} = 0 or 1, i= 1,2,……,m and j=1,2,……,n ……..[4]

Where a_j is the total cost available that worker j can be assigned.

3.2 Solution Procedure of NGAP

To solve NGAP first we have calculated the evaluation matrix for each alternative. Using the elements of Evaluation Matrix for alternatives Score function (S_{ij}) matrix has been calculated. Taking the Score function matrix (S_{ij}) as the initial input data we get the model 2.

Model 2.

Minimize $Z= \sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij}X_{ij}$ …….. [5]

s.t. the constraints [2], [3], [4].

To solve the model 2 we have used EDM and to verify it the problem has been transformed into LPP form and solved by LINGO 9.0.

3.3 Algorithm for NGAP

Step1. Construct the cost matrix of Neutrosophic generalized assignment problem $D = (C_{ij})_{m\times n}$

Step2. Determine the Evaluation Matrix of the job J_i as $E(J_i) = \left[T_{J_i}^{l}, T_{J_i}^{u}\right]$ where

$$[T_{J_i}^{l}, T_{J_i}^{u}]= \begin{bmatrix} \min((\frac{T_{J_{ij}}+I_{J_{ij}}}{2}),(\frac{1-F_{J_{ij}}+I_{J_{ij}}}{2})), \\ \max((\frac{T_{J_{ij}}+I_{J_{ij}}}{2}),(\frac{1-F_{J_{ij}}+I_{J_{ij}}}{2})) \end{bmatrix}$$

Step3. Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2(T_{J_{ij}}^{u} - T_{J_{ij}}^{l})$$

$$= 2\begin{bmatrix} \max((\frac{T_{J_{ij}}+I_{J_{ij}}}{2}),(\frac{1-F_{J_{ij}}+I_{J_{ij}}}{2})) - \\ \min((\frac{T_{J_{ij}}+I_{J_{ij}}}{2}),(\frac{1-F_{J_{ij}}+I_{J_{ij}}}{2})) \end{bmatrix}$$

Where $0 \le S(J_{ij}) \le 1$

Step4. Take the Score function matrix as initial input data for NGAP and solve it by EDM.

Step5. End.

4. Numerical Example

Let us consider a NGAP having four jobs and three machines where the cost matrix contains neutrosophic elements denoting time for completing jth job by ith machine. It is required to find optimal assignment of jobs to machines.

Input data table

$$\begin{array}{c} \\ J_1 \\ J_2 \\ J_3 \\ J_4 \end{array} \begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} [0.75,0.39,0.1] & [0.8,0.6,0.15] & [0.4,0.8,0.45] \\ [0.6,0.5,0.25] & [0.75,0.9,0.05] & [0.68,0.46,0.2] \\ [0.8,0.4,0.2] & [0.45,0.1,0.5] & [1.0,0.5,1.0] \\ [0.4,0.6,0.3] & [0.5,0.4,0.8] & [0.5,0.6,0.9] \end{bmatrix} \end{array}$$

Solution:

Evaluate $E(J_i)$ as the evaluation function of the job J_i as

$E(J_i) = \left[T_{J_i}^{l}, T_{J_i}^{u}\right]$ where

$$[T_{J_i}^{l}, T_{J_i}^{u}]= \begin{bmatrix} \min((\frac{T_{J_{ij}}+I_{J_{ij}}}{2}),(\frac{1-F_{J_{ij}}+I_{J_{ij}}}{2})), \\ \max((\frac{T_{J_{ij}}+I_{J_{ij}}}{2}),(\frac{1-F_{J_{ij}}+I_{J_{ij}}}{2})) \end{bmatrix}$$

Therefore elements of the Evaluation matrix for alternatives

$$\left[T^l_{J_{ij}}, T^u_{J_{ji}}\right] =$$

$$\begin{bmatrix} [0.57,0.645] & [0.7,0.725] & [0.6,0.675] \\ [0.55,0.625] & [0.825,0.925] & [0.57,0.63] \\ [0.6,0.6] & [0.275,0.3] & [0.25,0.75] \\ [0.5,0.65] & [0.3,0.45] & [0.35,0.55] \end{bmatrix}$$

Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2(T^u_{J_{ij}} - T^l_{J_{ij}})$$

$$= 2\begin{bmatrix} \max((\frac{T_{J_{ij}} + I_{J_{ij}}}{2}), (\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2})) - \\ \min((\frac{T_{J_{ij}} + I_{J_{ij}}}{2}), (\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2})) \end{bmatrix}.$$

Where $0 \le S(J_{ij}) \le 1$

Therefore elements of Score function matrix will be as follows-

$$S(J_{ij}) = \begin{bmatrix} 0.15 & 0.05 & 0.15 \\ 0.15 & 0.2 & 0.12 \\ 0.0 & 0.05 & 1.0 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}.$$

Solving $S(J_{ij})$ by EDM,

	M_1	M_2	M_3	Row Penalties
J_1	0.15	[0.05]	0.15	0.10
J_2	0.15	0.2	[0.12]	0.08
J_3	[0.0]	0.05	1.0	1.00
J_4	[0.3]	0.3	0.4	0.10
$a_j \rightarrow$	0.525	0.475	0.81	

Therefore optimal assignment is,

$J_1 \rightarrow M_2$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_1$, $J_4 \rightarrow M_1$.

To verify the problem, it has been transformed into LPP form and solved by LINGO 9.0 as follows-

Minimize Z=
$0.15x_{11}+0.05x_{12}+0.15x_{13}+0.15x_{21}+0.2x_{22}+0.12x_{23}+0.0x_{31}+0.05x_{32}+1.0x_{33}+0.3x_{41}+0.3x_{42}+0.4x_{43}$

s.t $\sum_{j=1}^{n} x_{ij} = 1$, i = 1, 2, ….. , m

$\sum_{i=1}^{m} c_{ij}x_{ij} \le a_j$, j = 1,2,…….,n

X_{ij} = 0 or 1, i= 1,2,……,m and j= 1,2,……,n

By LINGO 9.0, we get the solution as,

$x_{12}=1$, $x_{23}=1$, $x_{31}=1$, $x_{41}=1$

Therefore the optimal assignment is $J_1 \rightarrow M_2$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_1$, $J_4 \rightarrow M_1$.

Therefore the solution has been verified as same result has been obtained both by Model 1 and Model 2.

5. Conclusion

Neutrosophic set theory is a generalization of classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set because it not only considers the truth membership T_A and falsity membership F_A, but also an indeterminacy function I_A which is very obvious in real life situation. In this paper, we have considered the cost matrix as neutrosophic elements considering the restrictions on the available costs. By calculation Evaluation matrix and Score function matrix, the problem is solved by EDM which is very simple yet efficient method to solve GAP. Now to verify the solution the problem has been transformed to LPP form and solved by standard software LINGO 9.0.

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Neutrosophic Tangent Similarity Measure and Its Application to Multiple Attribute Decision Making

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Abstract: In this paper, the tangent similarity measure of neutrosophic sets is proposed and its properties are studied. The concept of this tangent similarity measure of single valued neutrosophic sets is a parallel tool of improved cosine similarity measure of single valued neutrosophic sets. Finally, using this tangent similarity measure of single valued neutrosophic set, two applications namely, selection of educational stream and medical diagnosis are presented.

Keywords: Tangent similarity measure, Single valued neutrosophic set, Cosine similarity measure, Medical diagnosis

1 Introduction

Smarandache [1, 2] introduced the concept of neutrosophic set to deal with imprecise, indeterminate, and inconsistent data. In the concept of neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and falsity-membership are independent. Indeterminacy plays an important role in many real world decision making problems. The concept of neutrosophic set [1, 2, 3, 4] generalizes the Cantor set discovered by Smith [5] in 1874 and introduced by German mathematician Cantor [6] in 1883, fuzzy set introduced by Zadeh [7], interval valued fuzzy sets introduced independently by Zadeh [8], Grattan-Guiness [9], Jahn [10], Sambuc [11], L-fuzzy sets proposed by Goguen [12], intuitionistic fuzzy set proposed by Atanassov [13], interval valued intuitionistic fuzzy sets proposed by Atanassov and Gargov [14], vague sets proposed by Gau, and Buehrer [15], grey sets proposed by Deng [16], paraconsistent set proposed by Brady [17], faillibilist set [2], paradoxist set [2], pseudoparadoxist set [2], tautological set [2] based on the philosophical point of view. From philosophical point of view, truth-membership, indeterminacy-membership, and falsity-membership of the neutrosophic set assume the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. Realizing the difficulty in applying the neutrosophic sets in realistic problems, Wang et al. [18] introduced the concept of single valued neutrosophic set (SVNS) that is the subclass of a neutrosophic set. SVNS can be applied in real scientific and engineering fields. It offers us additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information that manifest the real world. Wang et al. [19] further studied interval neutrosophic sets (INSs) in which the truth-membership, indeterminacy-membership, and false-membership were extended to interval numbers.

Neutrosophic sets and its various extensions have been studied and applied in different fields such as medical diagnosis [20, 21, 22, 23], decision making problems [24, 25, 26, 27, 28, 29, 30], social problems [31,32], educational problem [33, 34], conflict resolution [35, 36], image processing [37, 38, 39], etc.

The concept of similarity is very important in studying almost every scientific field. Literature review reflects that many methods have been proposed for measuring the degree of similarity between fuzzy sets studied by Chen [40], Chen et al., [41], Hyung et al.[42], Pappis & Karacapilidis [43], presented by Wang [44]. But these methods are not capable of dealing with the similarity measures involving indeterminacy. In the literature few studies have addressed similarity measures for neutrosophic sets and single valued neutrosophic sets [24, 45, 46, 47, 48, 49, 50, 51].

In 2013, Salama [45] defined the correlation coefficient, on the domain of neutrosophic sets, which is another kind of similarity measure. In 2013, Broumi and Smarandache [46] extended the Hausdorff distance to neutrosophic sets that plays an important role in practical application, especially in many visual tasks, computer assisted surgery, etc. After that a new series of similarity measures has been proposed for neutrosophic set using different approaches. In 2013, Broumi and Smarandache [47] also proposed the correlation coefficient between interval neutrosphic sets. Majumdar and Smanta [48] studied several similarity measures of single valued neutrosophic sets based on distances, a matching function, memebership grades, and entropy measure for a SVNS.

In 2013, Ye [24] proposed the distance-based similarity measure of SVNSs and applied it to the group decision making problems with single valued neutrosophic information. Ye [26] also proposed three vector similarity measure for SNSs, an instance of SVNS and interval valued neutrosophic set, including the Jaccard, Dice, and cosine similarity and applied them to multi-criteria decision-making problems with simplified neutrosophic information. Recently, Jun [51] discussed similarity measures on interval neutrosophic set based on Hamming distance and Euclidean distance and offered a numerical example of its use in decision making problems.

Broumi and Smarandache [52] proposed a cosine similarity measure of interval valued neutrosophic sets

Ye [53] further studied and found that there exsit some disadvantages of existing cosine similarity measures defined in vector space [26] in some situations. He [53] mentioned that they may produce absurd result in some real cases. In order to overcome theses disadvantages, Ye [53] proposed improved cosine similarity measures based on cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. In his study Ye [53] proposed medical diagnosis method based on the improved cosine similarity measures. Ye [54] further studied medical diagnosis problem namely,"Multi-period medical diagnosis using a single valued neutrosophic similarity measure based on tangent function`` However, it is yet to publish. Recently, Biswas et al. [50] studied cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. In hybrid environment Pramanik and Mondal [55] proposed cosine similarity measure of rough neutrosophic sets and provided its application in medical diagnosis. Pramanik and Mondal [56] also proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.

Pramanik and Mondal [57] proposed weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. Pramanik and Mondal [58] also proposed tangent similarity measures between intuitionistic fuzzy sets and studied some of its properties and applied it for medical diagnosis.

In this paper we have extended the concept of intuitionistic tangent similarity measure [56] to neutrosophic environment. We have defined a new similarity measre called "tangent similarity measure for single valued neutrosophic sets``. The properties of tangent similarity are established. The proposed tangent similarity measure is applied to medical diagnosis.

Rest of the paper is structured as follows. Section 2 presents preliminaries of neutrosophic sets. Section 3 is devoted to introduce tangent similarity measure for single valued neutrosophic sets and some of its properties. Section 4 presents decision making based on neutrosophic tangent similarity measure. Section 5 presents the application of tangent similarity measure to two problems namely, neutrosophic decision making of student's educational stream selection and neutrosophic decision making on medical diagnosis. Finally, section 6 presents concluding remarks and scope of future research.

2 Neutrosophic preliminaries

2.1 Neutrosophic sets

Definition 2.1[1, 2]

Let U be an universe of discourse. Then the neutrosophic set P can be presented of the form:

$P = \{< x:T_P(x), I_P(x), F_P(x)>, x \in U\}$, where the functions $T, I, F: U \rightarrow {}^{-}]0,1^{+}[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set P satisfying the following the condition.

$$^{-}0 \leq \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \leq 3^{+} \quad (1)$$

From philosophical point of view, the neutrosophic set assumes the value from real standard or non-standard subsets of ${}^{-}]0, 1^{+}[$. So instead of ${}^{-}]0, 1^{+}[$ one needs to take the interval [0, 1] for technical applications, because ${}^{-}]0, 1^{+}[$ will be difficult to apply in the real applications such as scientific and engineering problems. For two netrosophic sets (NSs), $P_{NS} = \{<x: T_P(x), I_P(x), F_P(x) > | x \in X\}$ and $Q_{NS} = \{< x, T_Q(x), I_Q(x), F_Q(x) > | x \in X\}$ the two relations are defined as follows:

(1) $P_{NS} \subseteq Q_{NS}$ if and only if $T_P(x) \leq T_Q(x)$, $I_P(x) \geq I_Q(x)$, $F_P(x) \geq F_Q(x)$

(2) $P_{NS} = Q_{NS}$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$

2.2 Single valued neutrosophic sets

Definition 2.2 [18]

Let X be a space of points (objects) with generic elements in X denoted by x. A SVNS P in X is characterized by a truth-membership function $T_P(x)$, an indeterminacy-membership function $I_P(x)$, and a falsity membership function $F_P(x)$, for each point x in X, $T_P(x)$, $I_P(x)$, $F_P(x) \in [0, 1]$. When X is continuous, a SVNS P can be written as follows:

$$P = \int_X \frac{<T_P(x), I_P(x), F_P(x)>}{x} : x \in X$$

When X is discrete, a SVNS P can be written as follows:

$$P = \sum_{i=1}^{n} \frac{<T_P(x_i), I_P(x_i), F_P(x_i)>}{x_i} : x_i \in X$$

For two SVNSs , $P_{SVNS} = \{<x: T_P(x), I_P(x), F_P(x)> | x \in X\}$ and $Q_{SVNS} = \{<x, T_Q(x), I_Q(x), F_Q(x)> | x \in X\}$ the two relations are defined as follows:

(1) $P_{SVNS} \subseteq Q_{SVNS}$ if and only if $T_P(x) \leq T_Q(x)$, $I_P(x) \geq I_Q(x)$, $F_P(x) \geq F_Q(x)$

(2) $P_{SVNS} = Q_{SVNS}$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$ for any $x \in X$

3 Tangent similarity measures for single valued neutrosophic sets

Let $P = <x(T_P(x)I_P(x)F_P(x))>$ and $Q = <x(T_Q(x), I_Q(x), F_Q(x))>$ be two single valued neutrosophic numbers. Now tangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as follows:

$$T_{SVNS}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}\left\langle 1 - tan\left(\frac{\pi}{12}\left(\left|T_P(x_i) - T_Q(x_i)\right| + \left|I_P(x_i) - I_Q(x_i)\right| + \left|F_P(x_i) - F_Q(x_i)\right|\right)\right)\right\rangle \quad (1)$$

Proposition 3.1. The defined tangent similarity measure $T_{SVNS}(A, B)$ between SVNS P and Q satisfies the following properties:

1. $0 \le T_{SVNS}(P, Q) \le 1$
2. $T_{SVNS}(P, Q) = 1$ iff $P = Q$
3. $T_{SVNS}(P, Q) = T_{NRS}(Q, P)$
4. If R is a SVNS in X and $P \subset Q \subset R$ then

$T_{SVNS}(P, R) \le T_{SVNS}(P, Q)$ and $T_{SVNS}(P, R) \le T_{SVNS}(Q, R)$

Proofs:

(**1**)

As the membership, indeterminacy and non-membership functions of the SVNSs and the value of the tangent function are within [0 ,1], the similarity measure based on tangent function also is within [0,1].

Hence $0 \le T_{SVNS}(P, Q) \le 1$

(**2**)

For any two SVNS P and Q if $P = Q$, this implies $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$. Hence

$\left|T_P(x) - T_Q(x)\right| = 0$, $\left|I_P(x) - I_Q(x)\right| = 0$, $\left|F_P(x) - F_Q(x)\right| = 0$,

Thus $T_{SVNS}(P, Q) = 1$

Conversely,

If $T_{SVNS}(P, Q) = 1$ then $\left|T_P(x) - T_Q(x)\right| = 0$, $\left|I_P(x) - I_Q(x)\right| = 0$, $\left|F_P(x) - F_Q(x)\right| = 0$ since tan(0)=0.

So we can we can write, $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, $F_P(x) = F_Q(x)$. Hence $P = Q$.

(**3**)

This proof is obvious.

(**4**)

If $P \subset Q \subset R$ then $T_P(x) \le T_Q(x) \le T_R(x)$, $I_P(x) \ge I_Q(x) \ge I_R(x)$, $F_P(x) \ge F_Q(x) \ge F_R(x)$ for $x \in X$.

Now we have the following inequalities:

$\left|T_P(x) - T_Q(x)\right| \le \left|T_P(x) - T_R(x)\right|$,

$\left|T_Q(x) - T_R(x)\right| \le \left|T_P(x) - T_R(x)\right|$;

$\left|I_P(x) - I_Q(x)\right| \le \left|I_P(x) - I_R(x)\right|$,

$\left|I_Q(x) - I_R(x)\right| \le \left|I_P(x) - I_R(x)\right|$;

$\left|F_P(x) - F_Q(x)\right| \le \left|F_P(x) - F_R(x)\right|$,

$\left|F_Q(x) - F_R(x)\right| \le \left|F_P(x) - F_R(x)\right|$.

Thus $T_{SVNS}(P, R) \le T_{SVNS}(P, Q)$ and $T_{SVNS}(P, R) \le T_{SVNS}(Q, R)$. Since tangent function is increasing in the interval $\left[0, \frac{\pi}{4}\right]$.

4. Single valued neutrosophic decision making based on tangent similarity measure

Let $A_1, A_2, ..., A_m$ be a discrete set of candidates, $C_1, C_2, ..., C_n$ be the set of criteria of each candidate, and $B_1, B_2, ..., B_k$ are the alternatives of each candidates. The decision-maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performances of candidates A_i (i = 1, 2,..., m) against the criteria C_j (j = 1, 2, ..., n). The values associated with the alternatives for MADM problem can be presented in the following decision matrix (see Table 1 and Table 2). The relation between candidates and attributes are given in the Table 1. The relation between attributes and alternatives are given in the Table 2.

Table 1: The relation between candidates and attributes

	C_1	C_2	...	C_n
A_1	d_{11}	d_{12}	...	d_{1n}
A_2	d_{21}	d_{22}	...	d_{2n}
...	...	...	...	...
A_m	d_{m1}	d_{m2}	...	d_{mn}

Table 2: The relation between attributes and alternatives

	B_1	B_2	...	B_k
C_1	δ_{11}	δ_{12}	...	δ_{1k}
C_2	δ_{21}	δ_{22}	...	δ_{2k}
...	...	...	...	...
C_n	δ_{n1}	δ_{n2}	...	δ_{nk}

Here d_{ij} and δ_{ij} are all single valued neutrosophic numbers.

The steps corresponding to single valued neutrosophic number based on tangent function are presented using the following steps.

Step 1: Determination of the relation between candidates and attributes

The relation between candidate A_i (i = 1, 2, ..., m) and

the attribute C_j (j = 1, 2, ..., n) is presented in the Table 3.

Table 3: relation between candidates and attributes in terms of SVNSs

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11}\rangle$	$\langle T_{12}, I_{12}, F_{12}\rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n}\rangle$
A_2	$\langle T_{21}, I_{21}, F_{21}\rangle$	$\langle T_{22}, I_{22}, F_{22}\rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n}\rangle$
...	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{1m1}\rangle$	$\langle T_{m2}, I_{m2}, F_{m2}\rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn}\rangle$

Step 2: Determination of the relation between attributes and alternatives

The relation between attribute C_i (i = 1, 2, ..., n) and alternative B_t (t = 1, 2, ..., k) is presented in the table 4.

Table 4: The relation between attributes and alternatives in terms of SVNSs

	B_1	B_2	...	B_k
C_1	$\langle T_{11}, I_{11}, F_{11}\rangle$	$\langle T_{12}, I_{12}, F_{12}\rangle$	...	$\langle T_{1k}, I_{1k}, F_{1k}\rangle$
C_2	$\langle T_{21}, I_{21}, F_{21}\rangle$	$\langle T_{22}, I_{22}, F_{22}\rangle$	...	$\langle T_{2k}, I_{2k}, F_{2k}\rangle$
...	...	...	...	...
C_n	$\langle T_{n1}, I_{n1}, F_{n1}\rangle$	$\langle T_{n2}, I_{n2}, F_{n2}\rangle$	...	$\langle T_{nk}, I_{nk}, F_{nk}\rangle$

Step 3: Determination of the relation between attributes and alternatives

Determine the correlation measure between the table 3 and the table 4 using $T_{SVNS}(P,Q)$.

Step 4: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of correlation measures. Highest value reflects the best alternative.

Step 5: End

5. Example 1: Selection of educational stream for higher secondary education (XI-XII)

Consider the illustrative example which is very important for students after secondary examination (X) to select suitable educational stream for higher secondary education (XI-XII). After class X, the student takes up subjects of his choice and puts focused efforts for better career prospects in future. This is the crucial time when most of the students get confused too much and takes a decision which he starts to dislike later. Students often find it difficult to decide which path they should choose and go. Selecting a career in a particular stream or profession right at this point of time has a long lasting impact on a student's future. If the chosen branch is improper, the student may encounter a negative impact to his/her carrier. It is very important for any student to choose carefully from various options available to him/her in which he/she is interested. So it is necessary to use a suitable mathematical method for decision making. The proposed similarity measure among the students' attributes and attributes versus educational streams will give the proper selection of educational stream of students. The feature of the proposed method is that it includes truth membership, indeterminate and falsity membership function simultaneously. Let $A = \{A_1, A_2, A_3\}$ be a set of students, B = {science (B_1), humanities/arts (B_2), commerce (B_3), vocational course (B_4)} be a set of educational streams and C = {depth in basic science and mathematics (C_1), depth in language (C_2), good grade point in secondary examination (C_3), concentration (C_4), and laborious (C_5)} be a set of attributes. Our solution is to examine the students and make decision to choose suitable educational stream for them (see Table 5, 6, 7). The decision making procedure is presented using the following steps.

Step 1: The relation between students and their attributes in the form SVNSs is presented in the table 5.

Table 5: The relation between students and attributes

Relation-1	C_1	C_2	C_3	C_4	C_5
A_1	(0.7, 0.3, 0.3)	(0.6, 0.3, 0.2)	(0.7, 0.1, 0.1)	(0.7, 0.4, 0.4)	(0.5, 0.3, 0.4)
A_2	(0.5, 0.2, 0.3)	(0.6, 0.2, 0.3)	(0.6, 0.1, 0.1)	(0.6, 0.3, 0.3)	(0.6, 0.3, 0.2)
A_3	(0.6, 0.2, 0.2)	(0.6, 0.3, 0.2)	(0.6, 0.1, 0.0)	(0.6, 0.3, 0.2)	(0.5, 0.3, 0.3)

Step 2: The relation between student's attributes and educational streams in the form SVNSs is presented in the table 6.

Table 6: The relation between attributes and educational streams

Relation-2	B_1	B_2	B_3	B_4
C_1	(0.9, 0.2, 0.2)	(0.7, 0.3, 0.2)	(0.8, 0.3, 0.3)	(0.4, 0.3, 0.6)
C_2	(0.6, 0.2, 0.2)	(0.8, 0.4, 0.2)	(0.4, 0.3, 0.5)	(0.3, 0.2, 0.5)

C_3	$\begin{pmatrix}0.7,\\0.2,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7,\\0.4,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.5,\\0.3,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.5,\\0.2,\\0.4\end{pmatrix}$
C_4	$\begin{pmatrix}0.8,\\0.2,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8,\\0.3,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.7,\\0.1,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.5,\\0.3,\\0.3\end{pmatrix}$
C_5	$\begin{pmatrix}0.7,\\0.3,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.6,\\0.3,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.8,\\0.2,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.7,\\0.2,\\0.1\end{pmatrix}$

Step 3: Determine the correlation measure between the table 5 and the table 6 using tangent similarity measures (equation 1). The obtained measure values are presented in table 7.

Table 7: The correlation measure between Reation-1(table 5) and Relation-2 (table 6)

Tangent similarity measure	B_1	B_2	B_3	B_4
A_1	0.91056	**0.91593**	0.87340	0.84688
A_2	**0.92112**	0.90530	0.90534	0.90003
A_3	**0.92124**	0.91588	0.87362	0.85738

Step 4: Highest correlation measure value of A_1, A_2 and A_3 are **0.91593**, **0.92112** and **0.92124** respectively. The highest correlation measure from the table 7 gives the proper decision making of students for educational stream selection. Therefore student A_1 should select in arts stream, student A_2 should select in science stream and student A_3 should select the science stream.

Example2: Medical diagnosis

Let us consider an illustrative example adopted from Szmidt and Kacprzyk [59] with minor changes. As medical diagnosis contains a large amount of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of a disease. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and falsity membership functions. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will give the proper medical diagnosis. The main feature of this proposed method is that it includes truth membership, indeterminate and false membership by taking one time inspection for diagnosis.

Now, an example of a medical diagnosis will be presented. Example: Let P = {P_1, P_2, P_3, P_4} be a set of patients, D = {Viral fever, malaria, typhoid, stomach problem, chest problem} be a set of diseases and S={Temperature, headache, stomach pain, cough, chest pain.} be a set of symptoms. The solution strategy is to examine the patient which will provide truth membership, indeterminate and false membership function for each patien regarding the relatiom between patient and different symptoms (see the table 8), the relation among symptoms and diseases (see the table 9), and the correlation measure between R-1 and R-2 (see the table 10).

Table 8: (R-1) The relation between Patient and Symptoms

R-1	Temper-ature	Headac he	Stom-ach pain	Cough	Chest-pain
P_1	$\begin{pmatrix}0.8,\\0.1,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.6,\\0.1,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.8,\\0.0\end{pmatrix}$	$\begin{pmatrix}0.6,\\0.1,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.6,\\0.3\end{pmatrix}$
P_2	$\begin{pmatrix}0.0,\\0.8,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.4,\\0.4,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.6,\\0.1,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.7,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.8,\\0.1\end{pmatrix}$
P_3	$\begin{pmatrix}0.8,\\0.1,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8,\\0.1,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.0,\\0.6,\\0.4\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.7,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.0,\\0.5,\\0.5\end{pmatrix}$
P_4	$\begin{pmatrix}0.6,\\0.1,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.5,\\0.4,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.3,\\0.4,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.7,\\0.2,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.3,\\0.4,\\0.3\end{pmatrix}$

Table 9: (R-2) The relation among symptoms and diseases

R-2	Viral fever	Malari a	Typhoi d	Stomac h proble m	Chest proble m
Temperatu re	$\begin{pmatrix}0.4,\\0.0,\\0.6\end{pmatrix}$	$\begin{pmatrix}0.7,\\0.0,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.3,\\0.3,\\0.4\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.7,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.8,\\0.1\end{pmatrix}$
Headache	$\begin{pmatrix}0.3,\\0.5,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.6,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.6,\\0.1,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.4,\\0.4\end{pmatrix}$	$\begin{pmatrix}0.0,\\0.8,\\0.2\end{pmatrix}$
Stomach pain	$\begin{pmatrix}0.1,\\0.7,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.0,\\0.9,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.7,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8,\\0.0,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.8,\\0.0\end{pmatrix}$

Cough	$\begin{pmatrix}0.4,\\0.3,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.7,\\0.0,\\0.3\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.6,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.7,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.8,\\0.0\end{pmatrix}$
Chest pain	$\begin{pmatrix}0.1,\\0.7,\\0.2\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.8,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.1,\\0.9,\\0.0\end{pmatrix}$	$\begin{pmatrix}0.2,\\0.7,\\0.1\end{pmatrix}$	$\begin{pmatrix}0.8,\\0.1,\\0.1\end{pmatrix}$

Table 10: The correlation measure between R-1 and R-2

Tangent similarity measure	Viral Fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	0.8522	**0.8729**	0.8666	0.6946	0.6929
P_2	0.7707	0.7257	0.8288	**0.9265**	0.7724
P_3	0.7976	0.7630	**0.8296**	0.7267	0.6921
P_4	**0.8469**	0.8407	0.7978	0.7645	0.6967

The highest correlation measure (shown in the Table 10) reflects the proper medical diagnosis. Therefore, patient P_1 suffers from malaria, P_2 suffers from stomach problem, and P_3 suffers from typhoid and P_4 suffers from viral fever.

Conclusion

In this paper, we have proposed tangent similarity measure based multi-attribte decision making of single valued neutrosophic set and proved some of its basic properties. We have presented two applications, namely selection of educational stream and medical diagnosis. The concept presented in this paper can be applied to other multiple attribute decision making problems in neutrosophic envirobment.

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Multi-objective non-linear programming problem based on Neutrosophic Optimization Technique and its application in Riser Design Problem

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Abstract- Neutrosophic set is a part of neutrosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework that has been recently proposed. The paper aims to give computational algorithm to solve a multi-objective non-linear programming problem (MONLPP) using neutrosophic optimization method. The proposed method is for solving MONLPP with single valued neutrosophic data. We made a comparative study of optimal solution between intuitionistic fuzzy and neutrosophic optimization technique. The developed algorithm has been illustrated by a numerical example. Finally, optimal riser design problem is presented as an application of such technique.

Keywords: Neutrosophic set, single valued neutrosophic set, neutrosophic optimization method, Riser design problem.

1 Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [1]. Since the fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy sets uses one real value $\mu_A(x) \in [0, 1]$ to represents the truth membership function of fuzzy set A defined on universe X. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [2] to capture the uncertainty of truth membership. In some applications we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov introduced the intuitionistic fuzzy sets [3], [5] which is a generalisation of fuzzy sets. The intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information. In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophy was introduced by Smarandache in 1995 [4]. The motivation of the present study is to give computational

algorithm for solving multi-objective non-linear programming problem by single valued neutrosophic optimization approach. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership functions in such optimization process and thus have made comparative study in intuitionistic fuzzy and neutrosophic optimization technique. Also as an application of such optimization technique optimal riser design problem is presented.

2 Some preliminaries

2.1 Definition -1 (Fuzzy set) [1]

Let X be a fixed set. A fuzzy set A of X is an object having the form $\tilde{A}$ = {(x,μ_A (x)), x ∈ X} where the function $\mu_A(x)$: X → [0, 1] define the truth membership of the element x ∈ X to the set A.

2.2 Definition-2 (Intuitionistic fuzzy set) [3]

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form $\tilde{A}^i = \{< X, \mu_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1] and $\nu_A(x)$: X→ [0, 1] define the Truth-membership and Falsity-membership respectively , for every element of x∈ X , $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

2.3 Definition-3 (Neutrosophic set) [4]

Let X be a space of points (objects) and $x \in X$. A neutrosophic set $\tilde{A}^n$ in X is defined by a Truth-membership function$\mu_A(x)$, an indeterminacy-membership function $\sigma_A(x)$ and a falsity-membership function $\nu_A(x)$ and having the form $\tilde{A}^n = \{< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X\}$.
$\mu_A(x), \sigma_A(x)\ and\ \nu_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. that is

$$\mu_A(x) : X \rightarrow\ ]0^-, 1^+[$$
$$\sigma_A(x) : X \rightarrow\ ]0^-, 1^+[$$
$$\nu_A(x) : X \rightarrow\ ]0^-, 1^+[$$

There is no restriction on the sum of $\mu_A(x), \sigma_A(x)\ and\ \nu_A(x)$, so

$$^-0 \leq \sup \mu_A(x) + \sup \sigma_A(x) + \sup \nu_A(x) \leq 3^+$$

2.4 Definition-3 (Single valued Neutrosophic sets) [6]

Let X be a universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< X, \mu_A(x), \sigma_A(x), \nu_A(x) > / x \in X\}$ where $\mu_A(x)$: X→ [0, 1], $\sigma_A(x)$: X→[0, 1] and $\nu_A(x)$: X→ [0, 1] with $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ for all x ∈ X.

Example Assume that X = [x_1, x_2, x_3]. X_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, $x_2$and x_3 are in [0, 1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a single valued neutrosophic set of X defined by

A = ⟨0.3,0.4,0.5⟩/x_1 + ⟨0.5,0.2,0.3⟩/x_2 + ⟨0.7,0.2,0.2⟩/x_3

2.5 Definition- 4(Complement): [6] The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by

$$\mu_{c(A)}(x) = \nu_A(x)$$

$$\sigma_{c(A)}(x) = 1 - \sigma_A(x)$$

$$\nu_{c(A)}(x) = \mu_A(x)$$

for all x in X.

Example 2: let A be a single valued

neutrosophic set defined in example 1. Then, c(A) = $\langle 0.5,0.6,0.3\rangle/x_1$ + $\langle 0.3,0.8,0.5\rangle/x_2$ +$\langle 0.2,0.8,0.7\rangle/x_3$.

2.6 Definition 5(Union):[6] The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A ∪ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_{c(A)}(x)$ = max $(\mu_A(x), \mu_B(x))$
$\sigma_{c(A)}(x)$=max($\sigma_A(x), \sigma_B(x)$)
$\nu_{c(A)}(x) =$ min$(\nu_A(x),$ $\nu_B(x))$ for all x in X

Example 3: Let A and B be two single valued neutrosophic sets defined in example -1. Then, A ∪ B = $\langle 0.6,0.4,0.2\rangle/x_1$ + $\langle 0.5,0.2,0.3\rangle/x_2$ + $\langle 0.7,0.2,0.2\rangle/x_3$.

2.7 Definition 6(Intersection):[6] The Intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A ∩ B, whose truth-membership, indeterminacy-membership and falsity-membership functions are are given by

$\mu_{c(A)}(x)$ = min $(\mu_A(x), \mu_B(x))$
$\sigma_{c(A)}(x)$ = min $(\sigma_A(x), \sigma_B(x))$

$\nu_{c(A)}(x)$ = max $(\nu_A(x), \nu_B(x))$ for all x in X

Example 4: Let A and B be two single valued neutrosophic sets defined in example -1. Then, A ∩ B = $\langle 0.3,0.1,0.5\rangle/x_1$ + $\langle 0.3,0.2,0.6\rangle/x_2$ + $\langle 0.4,0.1,0.5\rangle/x_3$.

Here, we notice that by the definition of complement, union and intersection of single valued neutrosophic sets, single valued neutrosophic sets satisfy the most properties of classic set, fuzzy set and intuitionistic fuzzy set. Same as fuzzy set and intuitionistic fuzzy set, it does not satisfy the principle of middle exclude.

3 Neutrosophic Optimization Technique to solve minimization type multi-objective non-linear programming problem.

A non-linear multi-objective optimization problem of the form

Minimize $\{f_1(x), f_2(x), \dots\dots\dots f_p(x)\}$ (1)

$g_j(x) \leq b_j$ j=1, ……..,q

Now the decision set $\widetilde{D}^n$, a conjunction of Neutrosophic objectives and constraints is defined as

$\widetilde{D}^n = (\bigcap_{k=1}^{p} \tilde{G}_k{}^n) \cap (\bigcap_{j=1}^{q} \tilde{C}_j{}^n) =$

$\{(x, \mu_{\widetilde{D}^n}(x), \sigma_{\widetilde{D}^n}(x), \nu_{\widetilde{D}^n}(x))\}$

Here $\mu_{\widetilde{D}^n}(x)$=min

$(\mu_{\tilde{G}_1{}^n}(x), \mu_{\tilde{G}_2{}^n}(x), \dots\dots\dots.\mu_{\tilde{G}_p{}^n}(x);$

$\mu_{\tilde{C}_1{}^n}(x), \mu_{\tilde{C}_2{}^n}(x), \dots\dots.\mu_{\tilde{C}_q{}^n}(x)$

for $all\ x \in X$.

$\sigma_{\widetilde{D}^n}(x)$=min

$(\sigma_{\tilde{G}_1{}^n}(x), \sigma_{\tilde{G}_2{}^n}(x), \dots\dots\dots.\sigma_{\tilde{G}_p{}^n}(x);$

$\sigma_{\tilde{C}_1{}^n}(x), \sigma_{\tilde{C}_2{}^n}(x), \dots\dots.\sigma_{\tilde{C}_q{}^n}(x))$

for all $x \in X$

$\nu_{\widetilde{D}^n}(x)$=max

$(\nu_{\tilde{G}_1{}^n}(x), \nu_{\tilde{G}_2{}^n}(x), \dots\dots\dots.\nu_{\tilde{G}_p{}^n}(x);$

$\nu_{\tilde{C}_1{}^n}(x), \nu_{\tilde{C}_2{}^n}(x), \dots\dots.\nu_{\tilde{C}_q{}^n}(x))$

for all $x \in X$.

Where $\mu_{\widetilde{D}^n}(x)$, $\sigma_{\widetilde{D}^n}(x)$, $\nu_{\widetilde{D}^n}(x)$ are Truth membership function, Indeterminacy membership function, falsity membership function of Neutrosophic decision set respectively. Now using the neutrosophic optimization the problem (1) is transformed to the non-linear programming problem as

Max α……(2)

Max γ

Min β

such that $\mu_{\tilde{G}_k{}^n}(x) \geq \alpha$,

$\mu_{\tilde{C}_j{}^n}(x \geq \alpha$

$\sigma_{\tilde{G}_k{}^n}(x) \geq \gamma$

$\sigma_{\tilde{C}_j{}^n}(x) \geq \gamma$

$\nu_{\tilde{G}_k{}^n}(x) \leq \beta$

$\nu_{\tilde{C}_j{}^n}(x) \leq \beta$

$\alpha + \beta + \gamma \leq 3$

$\alpha \geq \beta$

$\alpha \geq \gamma$

$\alpha, \beta, \gamma \in [0, 1]$

Now this non-linear programming problem (2) can be easily solved by an appropriate mathematical programming algorithm to give solution of multi-objective non-linear programming problem (1) by neutrosophic optimization approach.

4 Computational algorithm

Step-1: Solve the MONLP problem (1) as a single objective non-linear problem p times for each problem by taking one of the objectives at a time and ignoring the others. These solution are known as ideal solutions. Let x^k be the respective optimal solution for the k[th] different objective and evaluate each objective values for all these k[th] optimal solution.

Step-2: From the result of step-1, determine the corresponding values for every objective for each derived solution. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows.

$$\begin{bmatrix} f_1{}^*(x^1) & f_2(x^1) \dots\dots\dots f_p(x^1) \\ f_1(x^2) & f_2{}^*(x^2) \dots\dots\dots f_p(x^2) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ f_1(x^p) & f_2(x^p) \dots\dots\dots f_p{}^*(x^p) \end{bmatrix}$$

Step-3. For each objective $f_k(x)$, find lower bound $L_k{}^{\mu}$ and the upper bound $U_k{}^{\mu}$.

$U_k{}^{\mu}$ = max $\{f_k(x^{r*})\}$ and $L_k{}^{\mu}$ = min $\{f_k(x^{r*})\}$ where r =1, 2,…,k.

For truth membership of objectives.

Step-4. We represents upper and lower bounds for indeterminacy and falsity membership of objectives as follows:

$U_k{}^{\nu} = U_k{}^{\mu}$ and $L_k{}^{\nu} = L_k{}^{\mu}$ + t ($U_k{}^{\mu}$ - $L_k{}^{\mu}$)

$L_k{}^{\sigma} = L_k{}^{\mu}$ and $U_k{}^{\sigma} = L_k{}^{\mu}$ + s ($U_k{}^{\mu} - L_k{}^{\mu}$)

Here t and s are to predetermined real number in (0, 1).

Step-5. Define Truth membership, Indeterminacy membership, Falsity membership functions as follows:

$$\mu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \le L_k{}^{\mu} \\ \frac{U_k{}^{\mu} - f_k(x)}{U_k{}^{\mu} - L_k{}^{\mu}} & if\ L_k{}^{\mu} \le f_k(x) \le U_k{}^{\mu} \\ 0 & if\ f_k(x) \ge U_k{}^{\mu} \end{cases}$$

$$\sigma_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \le L_k{}^{\sigma} \\ \frac{U_k{}^{\sigma} - f_k(x)}{U_k{}^{\sigma} - L_k{}^{\sigma}} & if\ L_k{}^{\sigma} \le f_k(x) \le U_k{}^{\sigma} \\ 0 & if\ f_k(x) \ge U_k{}^{\sigma} \end{cases}$$

$$\nu_k(f_k(x)) = \begin{cases} 0 & if\ f_k(x) \le L_k{}^{\nu} \\ \frac{f_k(x) - L_k{}^{\nu}}{U_k{}^{\nu} - L_k{}^{\nu}} & if\ L_k{}^{\nu} \le f_k(x) \le U_k{}^{\nu} \\ 1 & if\ f_k(x) \ge U_k{}^{\nu} \end{cases}$$

Step-6. Now neutrosophic optimization method for MONLP problem gives a equivalent non-linear programming problem as:

Max α - β + γ …… (3)

Such that

$\mu_k(f_k(x)) \ge \alpha$

$\sigma_k(f_k(x)) \ge \gamma$

$\nu_k(f_k(x)) \le \beta$

α + β +γ ≤ 3

α ≥ β

α ≥ γ

α ,β, γ ∈ [0, 1]

$g_j(x) \le b_j$, x ≥ 0,

K=1,2,…….p;

j=1,2,……..q

Which is reduced to equivalent non-linear-programming problem as:

Max α - β + γ …… (4)

Such that

$f_k(x) + (U_k{}^{\mu} - L_k{}^{\mu}) .\alpha \le U_k{}^{\mu}$

$f_k(x) + (U_k{}^{\sigma} - L_k{}^{\sigma}) .\gamma \le U_k{}^{\sigma}$

$f_k(x) - (U_k{}^{\nu} - L_k{}^{\nu}) .\beta \le L_k{}^{\nu}$

for k = 1, 2, ……,p

α + β +γ ≤ 3

α ≥ β

α ≥ γ

α ,β, γ ∈ [0, 1]

$g_j(x) \le b_j$

for j=1,2,……..q.

x ≥ 0,

5 Illustrated example

Min $f_1(x_1, x_2) = x_1{}^{-1}x_2{}^{-2}$

Min $f_2(x_1, x_2) = 2\ x_1{}^{-2}x_2{}^{-3}$

Such that $x_1 + x_2 \le 1$

Here pay-off matrix is $\begin{bmatrix} 6.75 & 60.78 \\ 6.94 & 57.87 \end{bmatrix}$

Here $L_1{}^{\mu}$ =6.75, $U_1{}^{\nu} = U_1{}^{\mu}$ = 6.94 and $L_1{}^{\nu}$ = 6.75 + 0.19 t

$L_1{}^{\sigma} = L_1{}^{\mu}$ = 6.75 and $U_1{}^{\sigma}$ = 6.75 + 0.19 s

$L_2{}^{\mu}$ =57.87, $U_2{}^{\nu} = U_2{}^{\mu}$ = 60.78 and $L_2{}^{\nu}$ = 57.87 + 2.91 t

$L_2{}^{\sigma} = L_2{}^{\mu}$ = 57.87 and $U_2{}^{\sigma}$ = 57.87 + 2.91 s

We take t = 0.3 and s = 0.4

Table-1: Comparison of optimal solutions by IFO and NSO technique.

Optimization	Optimal	Optimal	Aspiration	Sum of

techniques	Decision Variables x_1^*, x_2^*	Objective Functions f_1^* , f_2^*	levels of truth, falsity and indeterminacy membership functions	optimal objective values
Intuitionistic fuzzy optimization(IFO)	0.3659009, 0.6356811	6.797078 58.79110	α^* = 0.719696 β^* = 0.022953	65.588178
Proposed neutrosophic optimization(NSO)	0.3635224, 0.6364776	6.790513 58.69732	α^*= 0.7156984 β^*= 0.01653271 γ^*= 0.2892461	65.487833

Table-1. Shows that Neutrosophic optimization technique gives better result than Intuitionistic fuzzy non-linear programming technique.

6 Application of Neutrosophic Optimization in Riser Design Problem

The function of a riser is to supply additional molten metal to a casting to ensure a shrinkage porosity free casting. Shrinkage porosity occurs because of the increase in density from the liquid to solid state of metals. To be effective a riser must solidify after casting and contain sufficient metal to feed the casting. Casting solidification time is predicted from Chvorinov's rule. Chvorinov's rule provides guidance on why risers are typically cylindrical. The longest solidification time for a given volume is the one where the shape of the part has the minimum surface area. From a practical standpoint cylinder has least surface area for its volume and is easiest to make. Since the riser should solidify after the casting, we want it's solidification time to be longer than the casting. Our problem is to minimize the volume and solidification time of the riser under Chvorinov's rule.

A cylindrical side riser which consists of a cylinder of height H and diameter D. The theoretical basis for riser design is Chvorinov's rule, which is $t = k\,(V/SA)^2$.

Where t = solidification time (minutes/seconds)

K = solidification constant for molding material (minutes/in^2 or seconds/cm^2)

V = riser volume (in^3 or cm^3)

SA = cooling surface area of the riser.

The objective is to design the smallest riser such that $t_R \geq t_C$

Where t_R = solidification time of the riser.

t_C = solidification time of the casting.

$$K_R\,(V_R/SA_R)^2 \geq K_C\,(V_C/SA_C)^2$$

The riser and the casting are assumed to be molded in the same material, so that K_R and K_C are equal. So $(V_R/SA_R) \geq (V_C/SA_C)$.

The casting has a specified volume and surface area, so $V_C/SA_C = Y$ = constant, which is called the casting modulus.

$(V_R/SA_R) \geq Y$, $V_R = \pi D^2H/4$, $SA_R = \pi DH + 2\pi D^2/4$

$(\pi D^2H/4)/(\pi DH + 2\pi D^2/4) = (DH)/(4H+2D) \geq Y$

We take V_c = 2.8.6=96 cubic inch. and SA_C = 2.(2.8+2.6+6.8)= 152 square inch.

then, $\frac{48}{19} D^{-1} + \frac{24}{19} H^{-1} \leq 1$

Multi-objective cylindrical riser design problem can be stated as :

Minimize $V_R(D, H) = \pi D^2H/4$

Minimize $t_R(D, H) = (DH)/(4H+2D)$

Subject to $\frac{48}{19}D^{-1} + \frac{24}{19}H^{-1} \leq 1$

Here pay-off matrix is $\begin{bmatrix} 42.75642 & 0.631579 \\ 12.510209 & 0.6315786 \end{bmatrix}$

Table-2. Values of Optimal Decision variables and Objective Functions by Neutrosophic Optimization Technique.

Optimal Decision Variables	Optimal Objective Functions	Aspiration levels of truth, falsity and indeterminacy membership functions
$D^* = 3.152158$ $H^* = 3.152158$	$V^*_R(D^*, H^*) = 24.60870$, $t^*_R(D^*, H^*) = 0.6315787$.	$\alpha^* = 0.1428574$ $\beta^* = 0.1428574$ $\gamma^* = 0.00001$

Conclusion: In view of comparing the Neutrosophic optimization with Intuitionistic fuzzy optimization method, we also obtained the solution of the numerical problem by Intuitionistic fuzzy optimization method [14] and took the best result obtained for comparison with present study. The objective of the present study is to give the effective algorithm for Neutrosophic optimization method for getting optimal solutions to a multi-objective non-linear programming problem. The comparisons of results obtained for the undertaken problem clearly show the superiority of Neutrosophic optimization over Intuitionistic fuzzy optimization. Finally as an application of Neutrosophic optimization multi-objective Riser Design Problem is presented and using Neutrosophic optimization algorithm an optimal solution is obtained.

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Abstract

This volume is a collection of fourteen papers, written by different authors and co-authors (listed in the order of the papers): F. Yuhua, K. Mandal, K. Basu, S. Pramanik, K. Mondal, S. Alkhazaleh, J. Nescolarde-Selva, J. L. Usó-Doménech, A. Betancourt-Vázquez, K. Pérez-Teruel, M. Leyva-Vázquez, A. Aydoğdu, I. Arockiarani, C. A. C. Sweety, F. Smarandache, L. Zhengda, S. Kar, S. Mukherjee, P. Das, and T. K. Kumar.

In first paper, the author studied Interpreting and Expanding Confucius' Golden Mean through Neutrosophic Tetrad. Hypercomplex Neutrosophic Similarity Measure & Its Application In Multicriteria Decision Making Problem is discussed in the second paper. Interval Neutrosophic Multi-Attribute Decision-Making Based on Grey Relational Analysis are studied in third paper. In fourth paper, More on Neutrosophic Norms and Conorms. Similarly in fifth paper, Ideological systems and its validation: a neutrosophic approach is discussed. In paper six, Modeling and analyzing non-functional requirements interdependencies with neutrosofic logic is studied by the author. On Entropy and Similarity Measure Interval Valued Neutrosophic Sets given in seventh paper. Topological structures of fuzzy neutrosophic rough sets are proposed in the next paper. Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies in the next paper. Further, Neutrosophic Decision Making Model for Clay-Brick Selection in Construction Field Based on Grey Relational Analysis is discussed by the authors in the tenth paper. In eleventh paper, Neutrosophy-based Interpretation of Four Mechanical Worldviews have been studied by the author. In the next paper, Application of Neutrosophic Set Theory in Generalized Assignment Problem is studied. In thirteenth paper, Tangent Similarity Measure and its application to multiple attribute decision making is discussed. At the end, Multi-objective non-linear programming problem based on Neutrosophic Optimization Technique and its application in Riser Design Problem are introduced by the authors.

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